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Mathematics. — *On the Limits of Holomorphic Functions.* By Prof. J. WOLFF. (Communicated by Prof. L. E. J. BROUWER).

(Communicated at the meeting of December 17, 1927).

FATOU's theorem: "if $f(z)$ is holomorphic and limited for $|z|<1$, $\lim_{\varrho \to 1} f(\varrho e^{\varphi i})$ exists, except perhaps for values of φ the set of which has the measure zero", has been completed by the brothers RIESS in the following way (Stokholm Congress 1916): "If a is an arbitrary number this limit is equal to a only for a φ-set of the measure zero".

In the following note we shall derive this result without making use of integrals of LEBESGUE.

Without loss of generality we may assume $f(0) \neq a$. We suppose $|f|<M$. We have for $0<\varrho<1$

$$2\pi \lg |f(0)-a| \leqq \int_0^{2\pi} \lg |f(\varrho e^{\varphi i})-a| \, d\varphi \quad . \quad . \quad . \quad . \quad (1)$$

Suppose $1>\varepsilon>0$ and let E_n be the set of intervals in which

$$\left| f\left\{\left(1-\frac{1}{n}\right)e^{\varphi i}\right\}-a\right|<\varepsilon.$$

If μE_n is the measure of E_n it follows from (1) and from $|f-a|<2M$, that

$$2\pi \lg |f(0)-a| \leqq \mu E_n \, . \, \lg \varepsilon + 2\pi \lg (2M),$$

hence $$\mu E_n \leqq \frac{C}{\lg \frac{1}{\varepsilon}}, \quad n = 1, 2, \ldots; \qquad C \text{ constant.}$$

Now the values of φ for which $\lim_{\varrho \to 1} f(\varrho e^{\varphi i}) = a$, belong to the limes inferior of E_n for $n \to \infty$.

Hence the measure of the set of these values is at most $\frac{C}{\lg \frac{1}{\varepsilon}}$ and as ε may be chosen arbitrarily between 0 and 1, this measure is zero.

Utrecht, December 6, 1927.

Geology. — *Measurements on slickensides on planes of stratification in folded regions.* By W. NIEUWENKAMP. (Communicated by Prof. L. RUTTEN.)

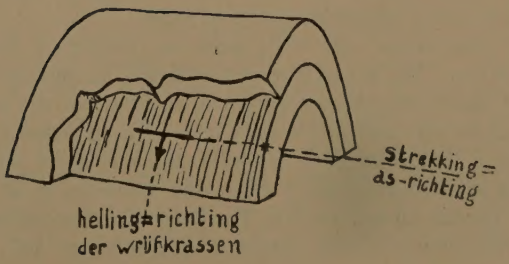
(Communicated at the meeting of January 28, 1928).

In folding the strata in the limbs of an anticline are pushed over each other. The younger strata push forward over the older ones toward the middle, toward the axis of the anticline. The truth of this statement is easily seen by every one who reproduces an anticline by means of a pack of cards. There is no reason why this movement should not in beds of suitable hardness, originate slickensides on the planes of stratification and as it is to be expected at right angles with the axis of the anticline the striae thereupon can furnish us an indication for the pitch of the latter. For when the axis of the fold is horizontal (parallel to the strike) the striae will follow the dip (Fig. A) when however the axis of the fold is inclined (when the fold has pitch) the striae will not follow the dip, deviating from it with an angle that increases with the amount of pitch (Fig. B).

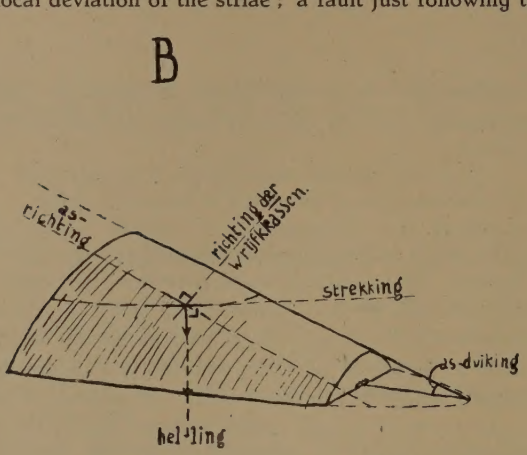
This would enable us to infer the pitch of the axis of folding from the direction of the striae in these slickensides. For if we draw on a plane of stratification (the axis runs parallel to the planes of stratification) a line at right angles with the striae, this line must be parallel to the axis of folding; so that by taking its inclination and azimuth we get pitch and direction of the latter.

In practice there are of course a number of causes that give to the striae

A



a direction differing from that required by this theory. Irregularities may engender a local deviation of the striae ; a fault just following the plane of



as-richting . . . . . = direction of the axis,
richting der wrijfkrassen = direction of the striae,
strekking . . . . . = strike,
helling . . . . . = dip,
as-duiking . . . . . = pitch.

stratification renders all our observations worthless for the object we have in view. Still it may be useful in our fieldwerk to have regard to the slickensides on the planes of stratification. An average of a number of observations of one and the same fold that do not differ too much, is surely to be relied upon.

This theory is to some extent verified by the students of geology of Utrecht, who have made in the Ardennes a fair number of observations of slickensides ; when several observations made in different parts of the same fold yielded the same result, this was always in complete accordance with the structural feature of the fold.

Mr. I. SWEMLE who worked out the geological structure of the broad strip of the Upper-Lower and Middle Devonian system between Ourthe and Lomme has on my request paid special regard to the slickensides on the planes of stratification, and has been so kind as to allow me to communicate some of his observations in this paper.

The first observation was made near the station of Jemelle in the sandstone of Co_1. The strata here strike N 65° E, and dip 75° S. The striae on the slickensides deviate eastward from the dip, the angle between the striae and the dip, measured here on four different slickensides, amounts to 9°. Hence a line on a plane of stratification at right angles with the striae has an azimuth of N 63° E. and dips westward at an angle of 9°.

South of Jemelle on the road to Forrières two slickensides have been measured with quite the same results on the southern limb of the same

anticline. The strike of the strata is here N 85° E, with a dip of 30° S. Again the striae deviate east of the direction of the dip at an angle of 30° ; thus giving for the axis an azimuth N 60° E, pitch 14° W.

Five hundred m. to the south of the preceding locality the beds strike N 75° E dip 40° S. The striae again deviate in the same direction at an angle of 17°, thus giving for the azimuth of the axis N 62° E, pitch 11° W.

These three observations all tend to show that the pitch of this anticline is about 11° in a direction S 60° W. This result is in complete agreement with the geological map.

Physiology. — *Radiated Vitamin B and Automatin action.* By Prof. H. ZWAARDEMAKER.

(Communicated at the meeting of January 28, 1928).

The oldest known vitamin is that of EIJKMAN, which protects from Beriberi, and has afterwards been termed Vitamin B. Its physiological effect, however, is still unknown. By mere accident I have been in a position to get acquainted with a special action exerted by the anti-Beriberi vitamin, an action which it ever possesses in a small degree, but which it obtains in a large measure through corpuscular radiation.

While searching for chemical bodies that might serve as mother-substance of our automatins, interesting myself in substances soluble in water and in alcohol, but not in ether, I was enabled to test JANSEN and DONATH's crystalline form of anti-beriberi vitamin. At first I had only a substance adsorbed to clay at my disposal delivered in the form of tabloids, of which usually 4 per day were given to a beriberi-patient; afterwards Prof. JANSEN sent me the approximately pure substance in ampoules of which 1 mgrm was injected intravenously in cases of acute beriberi. The ampoules contained hardly any potassium; the tabloids contained much of it, but by shaking them with alcohol of 96 %, only little potassium is transferred to the solution. In our experiments the concentration did, therefore, not exceed 4 mgr per litre.

In the first place I will demonstrate an ordinary automatin-effect with real automatin.

An eel's heart arrested by deprival of potassium, resumes its beats

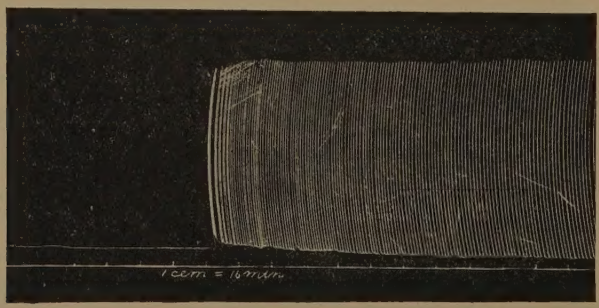


Fig. 1. An eel's heart brought to a standstill by deprival of potassium and automatin, recovers its pulsation through automatin from another heart.

through addition of the automatin derived from another radiated heart after a latency of 3½ hours. It will be seen that the frequency augments gradually, and has in this case reached its maximum after ¼ of an hour. This pulsation continues for 16 hours.

Next I will show the insignificant effect of non-radiated vitamin. Only separate beats, no quick rhythm appears.

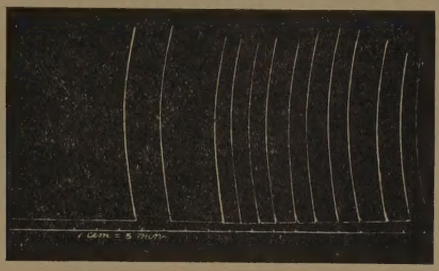


Fig. 2. Behaviour of a similar heart after adding non-radiated vitamine B to the circulating fluid.

This game is soon finished, after about two hours.

In the third place I will demonstrate the restoration of a test-heart by addition of radiated vitamin. After a latency of 1 hour intense contractions commence suddenly.

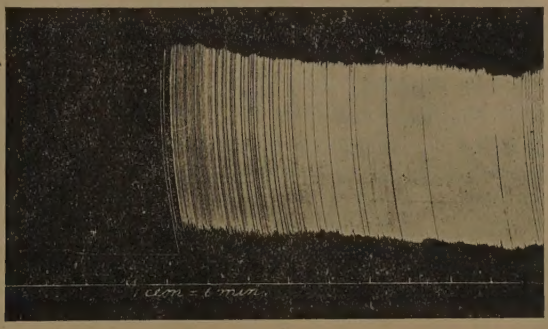


Fig. 3. Recovery of a similar heart after addition to the circulation of vitamin B that has been radiated beforehand during 15 hours.

The frequency gradually increases and reaches its maximum after 20 minutes. The regular beats continue for 13 hours.

Now it is clear that the form of the pulsations engendered by automatin are similar to those caused by vitamin B. In every experiment the latency depends on the time required for the preceding washing out of the potassium and automatin ; the frequency, on the other hand, depends on the

concentration of the stimulating substance, whether this be automatin or vitamin B activated by radiation.

In this respect the time for activation through corpuscular radiation should not be too short. When using the weak rays of 5 mgr of Radio-element in enamel, 2 hours will be too short, 4 hours just sufficient, 2×12 hours better than 12 hours.

It will be easily understood that these experiments do not point to any connection between the vitamin B originating from the vegetable kingdom and the automatinogen of the animals. Otherwise we should rather be inclined to believe that vitamin ingested together with the food is transmitted to the blood and is stored up everywhere in the skeletal musculature. In that case it would then be exposed to the radiation of the tissue potassium and partially become automatin, which in its turn stimulates the various automatic organs.

But we know already that there are more substances, which — while remaining inactive if not radiated — are activated through corpuscular radiation to such an extent that they produce a pronounced automatin-effect, just as well as the automatinogen and the vitamin B; those substances belong to the compounds supposed by JANSEN and DONATH to be allied to their crystalline substance.

It is impossible that this number may be increased. This may caution us to be careful, but it does not affect the main point, viz. the fact that the vitamin B, used at Batavia for practical therapy against Beriberi, acquires automatin-action through radiation. This opens up a broad field for study, for now of course also the other beriberi symptoms have to be studied.

Up to now the number of experiments with vitamin B is about 30. In half of them the vitamin was radiated, the other half were made without radiation. We ever found the sharp contrast as described. In the present investigation I could avail myself of Dr. ZEEHUISEN's kind assistance. I also consider it as a pleasant duty to thank Professors EIJKMAN and JANSEN and Dr. DONATH for placing their preparations at my disposal.

Experimental Phonetics. — *Contributions to an experimental investigation of the Dutch language. II. oo, eu and ee followed by r or not* [1]). By Miss L. KAISER. (Communicated by Prof. G. VAN RIJNBERK.)

(Communicated at the meeting of February 25, 1928).

Among vowels oo (o), *eu* (ø), and ee (e) take a peculiar position. This appears e.g. when we arrange the vowels in a so-called vowel-triangle: *aa* (*a*), *oe* (*u*), and *ie* (*i*) take the angles, oo, *eu* and *ee* being far away from them. The first mentioned are the so-called fundamental vowels, common to all primitive languages, the others are sounds that have only developed later. To pronounce oo, *eu* and *ee* the speech organs must assume a position that answers very definite requirements. To pronounce the so-called fundamental vowels, this is only the case to a less degree. In this connection it is worthwhile to compare the transition of oo, *eu* and *ee*, into the diphthongs *au* (ɔu), *ui* (ʌy) and *ei* (ɛi) in the careless pronunciation of the larger towns of Holland on the one hand as against the fact that in animal and instrumental sounds we may recognise *aa*, *oe* and *ie* often enough, but very seldom oo, *eu* and *ee*.

The influence of a following *r*-sound suffises to make these latter vowels lose their clearness. In Dutch in this case peculiar sounds are formed, which usually are compared or even identified with the short vowels: o, *u* and *i*. The vowels in *boor*, *beur* and *beer* are very different from those in *boot*, *beuk* and *beet*, and bear much resemblance to those in *bot*, *put* and *bit*, differing from these mainly or exclusively by their greater duration.

It is obvious that the way in which *r* itself is pronounced influences the phenomenon. To me it seems, that the altered vowels are most characteristic and constant before a lingual *r* (*r*), but that also with people who use a uvular *r* (*R*), this pronunciation is usual. Sometimes however, the vowels in that case are very much like the wide vowels o (ɔ), *u* (œ) and e (ɛ).

Concerning the articulation of *r* ZWAARDEMAKER [2]) stated, that in the pronunciation of *r* the muscles only fulfil the task of loosely raising the tongue, and that in the bottom of the mouth only then some tonus is noticeable, when a very strong relaxation has preceded. Hence after a vowel in which the bottom of the mouth is always tense, *r* must cause a relaxation. We may add that relaxation will take the more time and the more energy, as the vowel is pronounced with greater tension.

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[1]) From investigations made at the Physiological Laboratory of the Amsterdam University.

[2]) Nederl. Tijdschr. v. Geneesk. 1898. I. N⁰. 24.

As the characteristic tone of voiceless *r* STUMPF [1]) gives a^2—a^3 (870—1740 vibrations p.s.)

If we compare the accounts of different investigators concerning articulatory and acoustic qualities of *oo*, *eu* and *ee*, we find agreement on the following points. They are vowels pronounced with great tension in the speech organs, with a mid-position of the tongue (articulation for *oo* in the back-part of the mouth, for *eu* and *ee* in the front-part) and with a moderate opening of the jaws (varying little in the three sounds). The characteristic overtones are according to STUMPF [1]) :

oo	formant 388—517	
eu	underformant 388—517 ;	formant 1550—2324 ;
ee	underformant 388—517 ;	formant 1954—2764.

The few dates at hand concerning Dutch vowels (BOEKE, TER KUILE, BENJAMINS) agree very well with the above.

Experimental part.

A cinematographical record demonstrated that the outwardly visible differences between *oo*, *eu* and *ee*, followed by *r* or not, are very slight. In *oo* the opening of the mouth seems to be larger, in *ee* smaller when *r* follows. In *eu* no distinct difference could be noted. From records obtained by means of the labiograph of VON WILCZEWSKI the vertical distance between the lips was seen to be greater in *oo* and *eu* when *r* followed.

By means of ZWAARDEMAKER's apparatus to register speech movements (made of aluminium) records were made to compare the two series of sounds. As may be seen in fig. 1 the opening of the jaws was generally diminished when *r* followed. In *oo* and *eu* the difference was considerable, in *ee* small as in other vowels. This agrees well with EYKMAN's [2]) dates concerning the distance of the jaws.

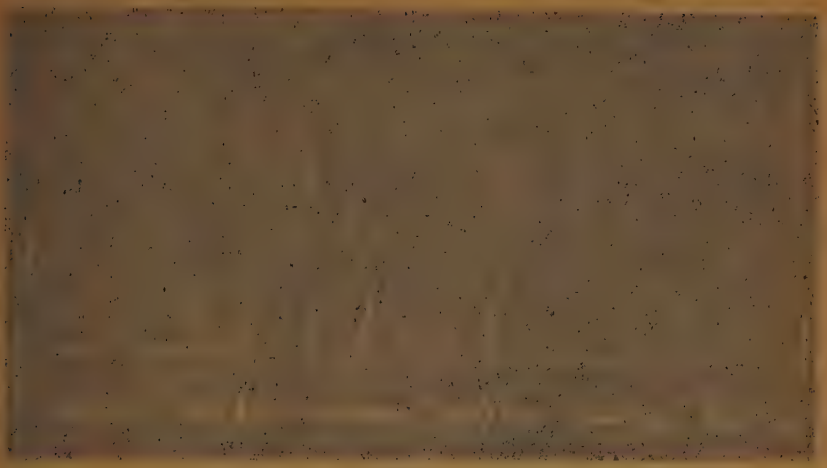
Palatograms in which by means of E. A. MEYER's method also the profil of the tongue was investigated, showed a picture as in fig. 2.

In all three vowels the tongue moves in a straight way, when *r* follows. Only a very small part of the mouthhole remains free from the moving tongue. No *r* following, the tongue moves in a curved way, the so-called front resonance-chamber remaining free. This curved way is only possible with a strong tension in the tongue. The larger palatal surfaces touched by the tongue, observed when *r* is following, may be due in first place to the smaller distance between the jaws.

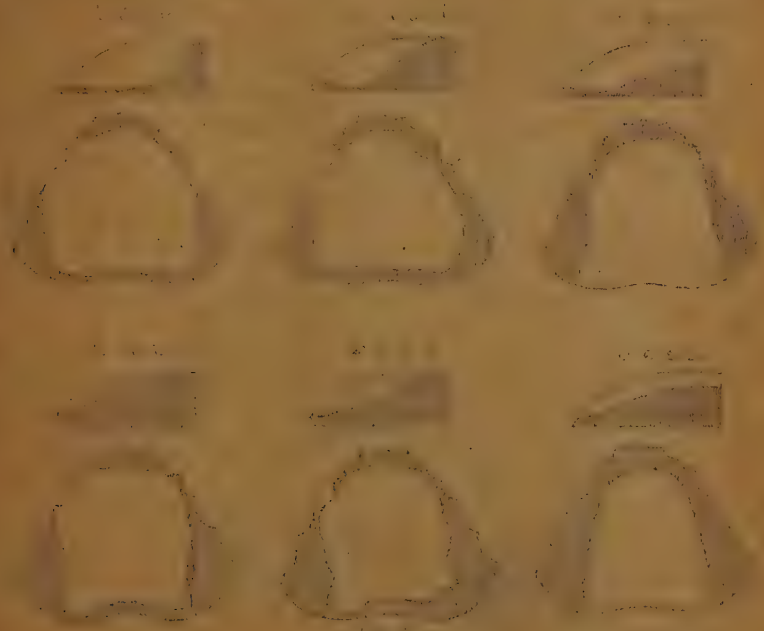
If we try to pronounce a pure *oo*, *eu* or *ee* before *r*, it appears to be impossible to make *r* follow the vowel immediately : between the two arises

[1]) C. STUMPF. Die Sprachlaute, 1926.

[2]) Onderzoekingen Physiol. Lab. te Utrecht. 5e reeks, II. 1901.



between the jaws and *a* st
c were clearly perceptible



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an *aa*-sound (compare English *poor, beer,* etc.). In the records made from such trials by means of ZWAARDEMAKER's apparatus, a great distance

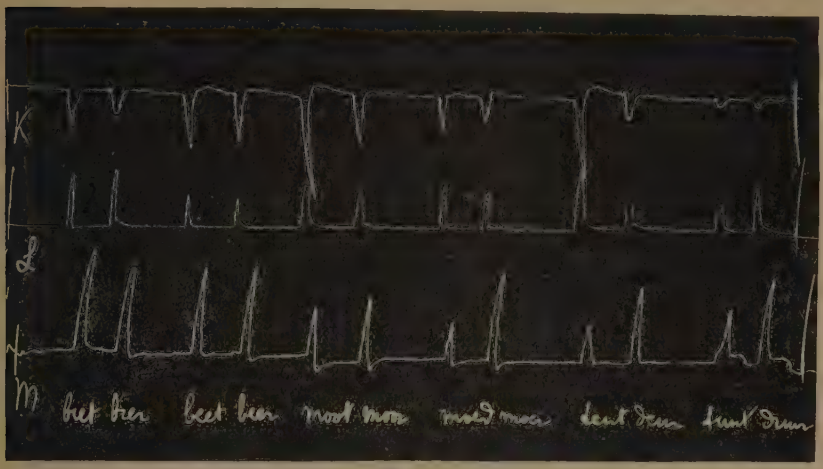

Fig. 1.

between the jaws and a strong relaxation of the bottom of the mouth before *r* were clearly perceptible.

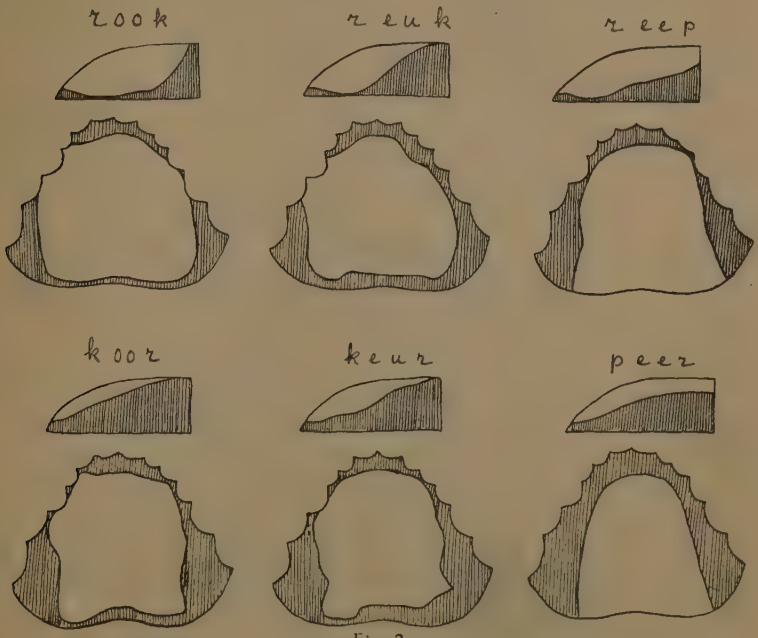

Fig. 2.

In deviating from the usual articulation it is possible, however, to pronounce a rather normal *oo*, *eu* or *ee* before *r*. To that effect the opening of the mouth must be actively or passively reduced in case of *oo*, and enlarged actively or passively in case of *eu* and *ee*.

A number of other influences make *oo*, *eu* and *ee* pass into the vowels that preceed *r*. Raising the point of the tongue, putting a piece of wood or wax in the front part of the mouth have almost the same effect. The other vowels remain practically unchanged thereby, as they do when followed by *r*.

To compare the acoustical qualities of the sounds, in the first place phonographical records were made, a lioretgraphe transcribing them ± 150 × enlarged on smoked paper. The greater part of these records were made from two subjects (B and K), both possessing a normal pronunciation with a lingual *r*. Fig. 3 shows some of these records. Accurate observing and simple measuring of these records showed the following: B spoke on a fundamental tone (pitch) of 170—190 p.s.; K on 200—225 p.s. The most important overtones were all within the limits indicated by STUMPF. For *oo* moreover a relatively strong tone of nearly 800 vibrations per second was found. This value agrees a.o. with that found by PAGET. In vowels not followed by *r* the second overtone was relatively strong especially in K, while in the vowels followed by *r* the importance of this overtone was of less importance, and became weak in B, while the amplitude of the third and to a less degree, that of the fourth overtone increased. The highest characteristic (formant) showed in *ee* a strong fall, under the influence of the following *r*, n.l. from 2300—2000.

For *eu*, where the highest characteristic was at 1600, a slight alteration in the same direction was noted. In *oo* the phenomenon was reversed, a feeble mounting of the overtone with frequency of 800 being noted. In all these cases, it was clearly visible that the highest characteristics towards the end of the vowel were transferred to a frequency of 1250, the formant of *r*. Records of the short vowels in *bot*, *dut* and *bit*, showed differences as well as striking similarity with both series. Determination of the first ten overtones by the method of HERMANN, which was applied in a few cases, affirmed the facts stated above.

The whispered vowels, too, were taken into account.

Also in whispering there is a marked difference between *oo*, *eu* and *ee* followed by *r* or not. Comparing also the sounds related to the investigated pairs of vowels with these latter, I came to the following series:

oe, *o* (ọ)[1], *oo* (*r*), *o* (ɔ), the higher characteristic rising and the lower being indistinct;

uu, *eu*, *eu* (*r*), *u*, *œ*, the higher characteristic falling and the lower rising;

ie, *ee*, *ee* (*r*), *i*, *e* (ɛ), the higher characteristic falling and the lower rising.

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[1]) KAISER. *The short o*. These Proceedings, Vol. 26, p. 745.

Fig. 3a.

Fig. 3b.

Fig. 3c.

However these results agree fairly well with those stated above, as a whole it did not appear quite satisfactory to me. Then BOUMAN [1]) developed a method to determine the overtones of vowels by means of electric resonance. The results of his investigations made a new explanation of facts acceptable.

Synopsis of results.

The acustical phenomena described above are corresponding in *eu* and *ee*, the reverse in *oo*. The cause of this can only be looked for in the place of articulation, which is the same for *eu* and *ee*, but different for *oo*. The consequence of this may be that in *oo* the narrowing of the opening of the mouth only reduces the space before the place of articulation, and in *eu* and *ee* especially the space behind it. This would imply that in *oo* the formant rises, in *eu* and *ee* the underformant rises, which is the case indeed.

The front resonance chamber in which the formant of *eu* and *ee*, is formed, becomes necessarily smaller with lessening of the distance between the jaws. The narrowing of the mouth opening attending it especially in *ee*, has a contrary effect and can be the stronger of the two influences at work, i.e. it can cause lowering of the formant : *eu* and *ee*. Widening of the mouth opening has in *oo* the same influence as reducement of the space : rising of the formant.

I cannot explain in details, at any rate not for *eu* and *ee*, how the moving point of the tongue and a small piece of wood or wax can have a similar effect. If BOUMAN's views are right, there is an explanation in them also for these phenomena.

Speaking generally we may say that *oo*, *eu* and *ee* are only pure under special circumstance and that everything which is anyhow detrimental to the pureness of these vowels, brings them in the class of the more or less imperfect vowels (BRÜCKE), where also the so-called short vowels have their place.

As *oo*, *eu* and *ee* are pre-eminently tense vowels and the pronunciation of *r* requires considerable laxity, it is comprehensible that they cannot be joined as they are.

In the Dutch language the joining of *r* to any other vowel is possible. To me this seems to prove that these latter vowels are not tense or at least far less so than *oo*, *eu* and *ee*.

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[1]) Archives Néerlandaises de phonétique expérimentale, 2, 1928.

Pathological Anatomy. — *Localisation of the tuberculous lesions in Javanese and Chinese inhabitants of Java.* By A. J. F. OUDENDAL.

(Communicated at the meeting of June 25, 1927)

In about 600 post-mortems of tuberculous patients of the Pathologic Anatomical Department of the S.T.O.V.I.A. at Weltevreden, we exactly registered where and which tuberculous manifestations were found. The collected data comprise:

295 Javanese men

85 Javanese women

208 Chinese men, who all had been suffering from some form of tuberculosis. Only part of the material was submitted to a systematic microscopical examination. I planned to test everything microscopically, but owing to retrenchments in the matter of assistants and subsidy, it became impossible to do so. Therefore only the results of the naked-eye research at the dissecting-table are mentioned.

Before giving a summary of the pathological appearances, I'll first relate something about the length, the weight and the age of the dissected corpses.

The average weight of the Javanese men was 34.1 kilograms $\left(\frac{9831.5}{288}\text{ kgs}\right)$, of the Javanese women 28 kgs $\left(\frac{2331.5}{83}\text{ kgs}\right)$ and of the Chinese men it was 33.7 kgs $\left(\frac{6914.52}{205}\text{ kgs}\right)$.

Concerning the average length, we found for the Javanese men 1.59 m. $\left(\frac{470.86}{295}\text{ m.}\right)$, the Javanese women 1.485 m. $\left(\frac{124.87}{84}\text{ m.}\right)$ and the Chinese men 1.61 m. $\left(\frac{334.50}{207}\text{ m.}\right)$.

Reliable informations about the ages is impossible to get here, so we only can speak about ages at an estimation. The average ages, thus estimated, are:

for the Javanese men 31 years

for the Javanese women 33 years

for the Chinese men 36 years

Scheme of the averages.

	Javanese men	Javanese women	Chinese men
Weight	34.1 kilograms	28 kilograms	33.7 kilograms
Length	1.59 meters	1.48[5] meters	1.61 meters
Approximate age	± 31 years	± 33 years	± 36 years

The oldest of the Javanese men was 75 years old, of the Javanese women 65 years, the Chinese men 70 years old. The youngest ones were respectively 15, 18 and 16 years old. There was one Chinese child of 9 months old.

We obtained our information from the notes on 2418 post-mortems, collected in about 5½ years. Year by year the proportion of the tuberculous cases in regard to all the post-mortems, showed us for the :

	T.B.C.+	Total numbers of post-mortems	Percentage
1st year (1921—'22)	102	597	16.7 %
2nd year (1922—'23)	141	608	23.2 „
3rd year (1923—'24)	136	397	34.3 „
4th year (1924—'25)	107	364	29.4 „
5th year (1925—'26)	96	331	29 „
6th year (1926—'27 half)	42	121	34.7 „

In the table above all the patients, who died of tuberculosis are counted, so also the Chinese women, european and arabian men and women.

Taking 2418 post-mortems as a whole we found among this number 624 post-mortems of tuberculous patients, or a quarter of the total amount.

As a rule we seldom dissect any child, so about the localisation of the tuberculous lesions in babies and very young people we cannot make any statements. Among the Chinese male patients there was one baby of 9 month old, suffering from bilateral closed tuberculosis of the lungs, with a tuberculosis of the bronchial glands. How the tuberculous inflammations are localized in the body, of the different groups (Javanese men, Javanese women and Chinese men) may be found in the adjoining lists.

Javanese men.

We made 295 post-mortems of native men, and found about the lungs that :

177 × the right lung showed one or more cavities

173 × the left lung did so and

118 × these cavities were found in both lungs at the same time, this makes 68 % of the greatest possible amount (173).

On one side the cavities were found 59 × in the right lung and 56 × in the left lung.

Tuberculosis without cavities was found in 90 cases in the right as well as in the left lung. Bilateral "closed" tuberculosis was to be found in 30 cases (33 %), so 60 times we found a tuberculosis of one lung only without any cavities.

Cavities of the right lung were combined with a closed tuberculosis of

the left lung in 48 cases, in 11 cases cavities of the right lung were combined with left lung free from tuberculosis.

Cavities of the left lung were combined with a closed right tuberculosis 52 times, and only 4 times there were left lung-cavities with a "free" right lung. The cavities were mostly found in the cranial part of the lungs, also in Javanese women and Chinese men.

Of the 295 post-mortems :

28 × the right lung was free from tuberculous lesions,

31 × the left lung remained free,

12 × both lungs were free from tuberculosis. Those 12 pairs of "free" lungs were found in :

4 men with caries of the lumbal spine,

1 man with caries of the thoracal spine,

3 men only showed tuberculosis of the bronchial glands,

2 men with a coxitis tuberculosa sinistra,

1 woman with a gonitis tuberculosa dextra,

1 man only showed a tuberculous inflammation of the mesenterial glands.

About the tuberculous lymphadenitis we found :

In 170 out of 295 Javanese men tuberculosis of the bronchial glands (in 57½ %) of the total amount, 88 times (in 29 % of the amount) tuberculosis of the mesenterial glands, and 64 patients with tuberculosis of both glands (22 % of the total amount).

In Javanese men tuberculous bronchial glands were found in :

49 % with bilateral lung cavities,

17 % with cavities of the right lung,

17 % with cavities of the left lung,

9 % with bilateral closed tuberculosis.

Tuberculous mesenterial glands we found in :

52 % with ulcers all along the intestines,

18 % with ulcers in the colon only,

12 % with ulcers in the small intestine only,

17 % without ulcers visible to the naked eye in any part of the intestine.

In 55 out of the same 295 men, we found tuberculous ulcers on the vocal chords, the epiglottis etc (18 %).

These pharynxulcers we found :

29 × with bilateral lung cavities	53 %
15 × with cavities of the left lung	27 %
9 × with cavities of the right lung	16 %
1 × with bilateral closed tuberculosis	2 %
1 × with lungs, free from tuberculosis	2 %

Phneumothorax on one side was found :

4 × at the left lung,

3 × at the right lung.

Menigitis tuberculosa was found in 19 cases (5½ %).

Looking for lesions of the intestines, it struck me how many tuberculous ulcers were found, and very often accompanied by an appendicitis tuberculosa chronica too. Therefore I should like to have an answer to the following questions: Is there any connection between these tuberculous ulcers of the intestines and the great quantity of parasites (hookworms, amoebes), we find here? And isn't it necessary with patients with a tuberculosis of the lungs or the bones combined with dysenterical troubles, to think more and more about a tuberculous enteritis or colitis? Also for practical purposes with a view to the therapeutics this must be an interesting point.

Of the 295 Javanese men there were:

131 with tuberculous ulcers of the colon 45 %
100 with tuberculous ulcers of the small intestine 34 %
142 didn't show any ulcers of the intestines 47 %

In 79 cases the whole of the intestines showed tuberculous ulcera, so there were:

21 men with ulcers in the small intestine only,
52 men with ulcers in the colon only.

Again there is a visible connection between the open tuberculosis of the lungs and the frequency of the tuberculosis of the intestines. So in 79 cases with tuberculous ulcers all along the intestines, more than half (43 cases) are combined with cavities in both lungs, 14 with cavities in the left lung, and 17 with cavities in the right lung, so from these 79 patients with tuberculosis of the tractus intestinalis, 74 were suffering from an infectious form of "open" tuberculosis of the lung. Only in 5 cases we couldn't find any cavities on examination with the naked eye.

On the other hand it is interesting to note that:

118 corpses with bilateral cavities in the lungs, showed:

43 × tuberculosis of the whole intestines,
20 × tuberculosis of the colon only,
3 × tuberculosis of the small intestine only, so 52 of these cases were free from visible ulcers of the intestines.

We found 12 cases of tuberculosis of the bones:

6 × Caries of the lumbar spine,
5 × Coxitis tuberculosa,
2 × Gonitis tuberculosa,
2 × Caries of the ribs,
1 × Caries of the thoracal spine.

Javanese women.

In 85 Javanese women, we found 36 × one or more cavities of the lungs, as well on the right as on the left hand side. In 24 corpses it was found to be bilateral, follows that cavities on *one* side only were found in 12 cases (right and left).

In 42 right lungs and 38 left lungs we found tuberculous manifestations without cavities, but in 7 corpses the right lung, and 11 times the left lung remained absolutely free from visible tuberculous lesions. Only 3 times *both* lungs seemed to be absolutely free from tuberculosis.

These 3 pairs of free lungs were combined:

1 × with Caries caput femoris dextra,

1 × with Gonitis tuberculosa dextra,

1 × Caries ossis pubis.

We never found a tuberculous inflammation of the bronchial glands or the mesenterical glands only.

Of the lung cavities on one-side 12 × cavities of the left lung were combined with a tuberculosis of the right lung (without cavities) and of the 12 cases with cavities of the right lung 10 were combined with tuberculosis of the left lung without cavities only, and 2 with a "free" left lung.

A bilateral tuberculosis of the lungs without cavities was found in 24 corpses.

We regularly found tuberculous glands; out of 85 women 40 female showed a tuberculosis of the bronchial glands (47 %), and in 36 cases (41 %) there was a tuberculous inflammation of the mesenterial glands. In 20 cases the bronchial as well as the mesenterial glands were altered.

Of the 40 tuberculous bronchial glands we found:

32½ % with cavities on both sides,

10 % with cavities on the right hand side,

10 % with cavities on the left hand side,

35 % with bilateral tuberculosis of the lungs (without cavities).

The 36 cases of tuberculosis of the mesenterial glands were distributed as follows:

55½ % with tuberculous ulcers all along the intestines,

19 % with tuberculous ulcers in the small intestine only,

5½ % with tuberculous ulcers in the colon only,

19 % without any sign of intestinal tuberculosis.

In 10 women we found a tuberculosis of the larynx, epiglottis, etc. (11 %), always with an open tuberculosis of the lungs with cavities.

5 out of ten with cavities on both sides,

4 with cavities of the right lung,

1 with cavities of the left lung.

No phneumothorax was found in those 85 Javanese women.

About the tuberculosis of the intestines in 31 cases we found tuberculous ulcers all along the intestines, 10 times in the small intestine only, and 12 times in the colon only, respectively in 36½ %, 14 % and 12 %. And 37½ % of the intestines were free from tuberculosis.

These 31 cases of tuberculosis all along the intestines were in 10 cases combined with cavities in both lungs, 8 times with cavities in one lung, and 10 times with a tuberculosis of the lungs without cavities.

Menigitis was found in 5 corpses, tuberculosis of the bones in 6, so :
2 × Gonitis tuberculosa dextra,
1 × Tuberculosis of the right elbow,
2 × Caries of the thoracal spine,
1 × Caries caput femoris,
1 × Caries ossis pubis,
1 × Caries of the ribs.

Chinese men.

In 208 postmortems of Chinese men, we found :

129 × cavities in the right lung, 117 × cavities in the left lung, and in 89 cases we found cavities in both lungs. There were 68 right- and 76 left lungs which showed tuberculous lesions without cavities. In 37 cases there was an inflammation of both lungs, and only 4 Chinese were in possession of both lungs free from tuberculosis, although in 11 cases the right lung and in 15 cases the left lung was free from tuberculosis.

The 4 pairs of "free" lungs were combined with :

2 cases of caries of the spine,

1 case of tuberculosis of the bronchial glands,

1 case of tuberculosis of the bronchial glands with ulcers in the small intestine.

In 40 cases of cavities in the right lung only, 35 were combined with a tuberculosis of the left lung without cavities, so 5 were found in combination with a left lung free from tuberculous lesions.

Out of 28 cases with cavities in the left lung, we found 25 right lungs without cavities, and 3 "free" right lungs.

Tuberculous glands were very often found, in 208 postmortems we found 97 × tuberculous bronchial glands (41 %) and 66 × tuberculous mesenterial glands (31 %).

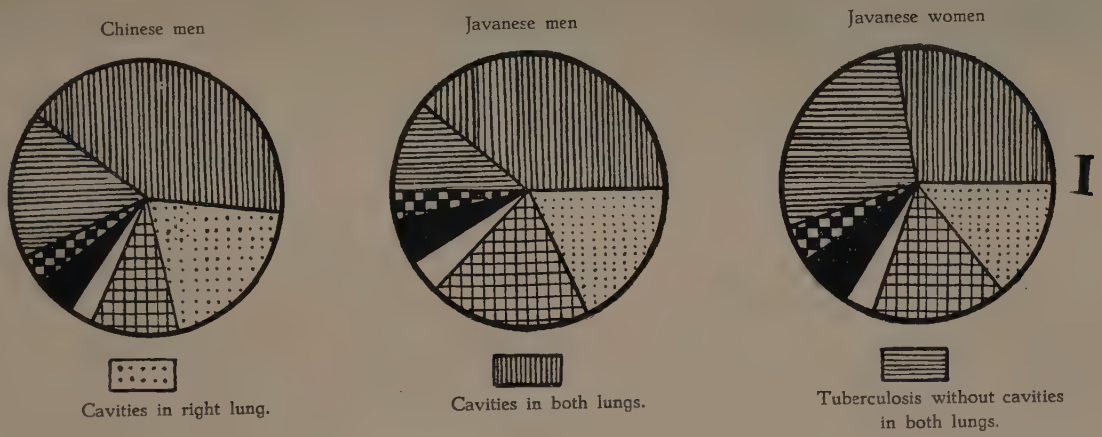
46 patients possessed tuberculous bronchial as well as mesenterial glands, so in 22 % of the total amount of Chinese corpses.

Tuberculous bronchial glands were found in :
54½ % with bilateral cavities of the lungs,
19½ % with cavities of the right lung,
10 % with cavities of the left lung,
12½ % with a bilateral tuberculosis of the lungs without cavities.

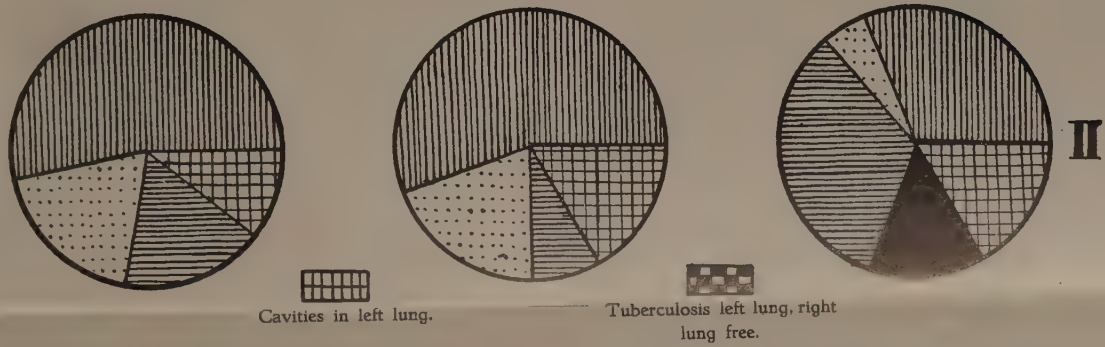
Tuberculous mesenterial glands were found in :
51 % with ulcers all along the intestines,
6 % with ulcers in the small intestine only,
22½ % with ulcers in the colon only,
20 % without any visible lesions of the intestines.

We found 39 cases of tuberculous ulcers of larynx and epiglottis :

A. J. F. OUDENDAL: PROPORTIONS OF TUBERCULOUS MANIFESTATIONS IN 600 JAVANESE AND CHINESE INHABITANTS OF JAVA.



Proportions of the different manifestations of tuberculosis of the lung.



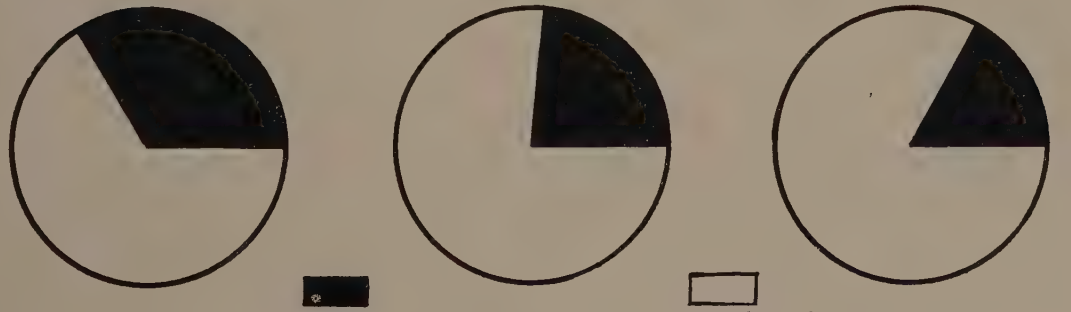
Tuberculous ulcers all along the intestins compared to the different forms of tuberculosis of the lung.

Tuberculosis in whole intestines.

Intestines free from tuberculosis.

V

Tuberculous ulcers all along the intestins, the colon only and the small intestine only compared to the cases without any intestinal tuberculosis.



Postmortens with tuberculosis.

Postmortens without tuberculosis.

VI

Proportions of postmortens with tuberculous affections and postmortens free from tuberculosis.

21 × with bilateral cavities (54 %),
5 × with cavities in the left lung (13 %),
10 × with cavities in the right lung (25 %).

A pneumothorax was found 5 × on the left and 2 × on the right hand side.

Menigitis tuberculosa was found in 12 corpses (6 %).

Tuberculous ulcers all along the intestines was seen **66** ×, in the small intestine only **28** ×, and in the colon only **20** ×, so out of 208 patients only 94 (45 %) were free from tuberculosis of the intestines.

Out of the 66 cases with tuberculosis of the tractus intestinalis 56 were combined with lung cavities (85 %), but only $^1/_3$ of the cases with bilateral lungcavities were free from tuberculosis of the intestines, so out of 89 patients with bilateral lung cavities there were :

35 suffering from tuberculosis all along the intestines,
11 suffering from tuberculous ulcers in the small intestine only,
16 suffering from tuberculous ulcers in the colon only.

Tuberculosis of the bones we found 9 × in 7 Chinese men.

4 × Caries of the lumber spine,
2 × Tuberculosis of the kneejoint,
1 × Tuberculosis of the foot,
1 × Tuberculosis of the sternum,
1 × Coxitis tuberculosa.

We very seldom come across a miliair tuberculosis here. Perhaps according to the fact that we only get very few postmortems of young people. We only found it :

6 × in Javanese men 2 %,
3 × in Javanese women 4 %,
3 × in Chinese men 1½ %.

Pleuritis chronica adhaesiva is seen in about every corps. The amount of corpses without this pleuritis I estimate to be less than 1 %.

As a matter of fact this short summary cannot give you an idea about the dissemination of tuberculous diseases amongst the *living* population.

The distribution of the tuberculous lesions, as we saw them at the postmortems, cannot give us any certainty about the distribution of the tuberculous lesions as they were during life.

So this summary only has a limited value, and is only meant as a beginning of large statistics, more so as the small groups and the impossibility of having everything microscopically tested, is a definite reason to refrain from drawing general conclusions.

Short summary of the notes on 295 Javanese men.

I. Right lung with cavities : 177 × ; left lung : 173 ×
Right lung with tuberculosis without cavities : 90 × } left lung : 90 ×
Right lung without tuberculous lesions : 28 × } left lung : 31 ×
Bilateral cavities : 118 ×
Bilateral tuberculosis of the lung without cavities } : 30 ×
Both lungs free from tuberculosis : 12 ×

II. Tuberculous bronchial glands : 170 ×
Tuberculous mesenterial glands : 88 ×
Tuberculous ulcers in the small intestine : 100 ×
Tuberculous ulcers in the colon : 131 ×
Tuberculous ulcers on the larynx : 55 ×

III. Tuberculosis of the peritoneum : 20 ×
Tuberculosis of the liver : 35 ×
Tuberculosis of the spleen : 47 ×
Tuberculosis of the kidneys : 44 ×
Menigitis tuberculosa : 19 ×
Phneumothorax left lung : 4 ×
Phneumothorax right lung : 3 ×

IV. With 12 pairs of "free" lungs, we found :
4 × Caries of the lumbar spine,
1 × Caries of the thoracal spine,
2 × Coxitis tuberculosa sinistra,
1 × Gonitis tuberculosa dextra,
1 × Tuberculosis of the mesenterial glands only,
3 × Tuberculosis of the bronchial glands only.

V. Tuberculous ulcers all along the intestines : 79 ×
Tuberculous ulcers in the small intestine *only* : 21 ×
Tuberculous ulcers in the colon *only* : 52 ×

VI. In 118 patients with bilateral lung cavities we found :
43 × tuberculous ulcers in the whole bowels,
20 × tuberculous ulcers in the colon *only*,
3 × tuberculous ulcers in the small intestine *only*,
52 × no ulcers.

VII. Of 30 patients with bilateral tuberculosis of the lungs without cavities we found :
5 with tuberculous ulcers all along the intestines,
1 with tuberculous ulcers in the small intestine *only*,
6 with tuberculous ulcers in the colon *only*,
18 without any intestinal ulcers.

VIII. In 59 cases of right lung-cavities only, we found :
17 × tuberculous ulcers all along the intestines,
6 × tuberculous ulcers in the small intestine *only*,
8 × tuberculous ulcers in the colon *only*,
28 × no ulcers.

IX. In 56 cases of left lung-cavities only, we found :
14 × tuberculous ulcers all along the intestines,
5 × tuberculous ulcers in the small intestine *only*,
11 × tuberculous ulcers in the colon *only*,
26 × no ulcers.

X. In 12 cases of bilateral "free" lungs, we found :
5 × tuberculous ulcers in the colon *only*,
2 × tuberculous ulcers in the small intestine *only*,
5 × no ulcers.

XI. Summary of the lungs.

Bilateral cavities	: 118 ×			
Bilateral tuberculosis without cavities	: 30 ×			
Bilateral "free" lungs	: 12 ×			
Cavities of the right lung, tuberculosis of the left „	: 41 ×	Cavities of the left lung, tubercul. of the right „	:	52 ×
Cavities of the right lung, free left lung	: 11 ×	Cavities of the left lung, free right lung	:	4 ×
Free right lung, Tubercul. of the left lung	: 12 ×	Free left lung, Tuberc. of the right lung	:	8 ×

XII. Out of 55 cases of tuberculosis of the larynx there were :
29 with bilateral lung cavities,
9 with cavities of the right lung,
15 with cavities of the left lung,
1 with bilateral tuberculosis of the lungs without cavities,
1 with absolutely free lungs.

XIII. We also found :
3 × tuberculosis of the testes,
2 × tuberculosis of the bladder,
11 × tuberculosis of the prostatus,
3 × tuberculosis of the epididymis,
6 × tuberculosis of the suprarenal body,
1 × tuberculosis of the pancreas,
1 × tuberculosis of the thyroid gland,
6 × tuberculosis of the lumbar spine,
1 × tuberculosis of the thoracal spine,
2 × tuberculosis of the ribs,
5 × coxitis tuberculosa,
2 × tuberculosis of the kneejoint

Summary of 85 Javanese women.

I. Cavities of the right lung : 36 ×; of the left lung : 36 ×
Tuberculosis of the right lung without cavities : 42 ×; left lung : 38 ×
"Free" right lungs : 7 ×; left lung : 11 ×
Bilateral lung-cavities : 24 ×
Bilateral tuberculosis of the lungs without cavities : 24 ×
Bilateral "free" lungs : 3 ×

II. Tuberculosis of the bronchial glands : 40 ×
Tuberculosis of the mesenterial glands : 36 ×
Tuberculous ulcers in the small intestine : 41 ×
Tuberculous ulcers in the colon : 43 ×
Tuberculous ulcers of the larynx etc. : 10 ×

III. Tuberculosis of the peritoneum : 7 ×
Tuberculosis of the liver : 11 ×
Tuberculosis of the spleen : 17 ×
Tuberculosis of the kidneys : 18 ×
Menigitis tuberculosa : 5 ×

IV. With 3 pairs of "free" lungs we found :
1 × gonitis tuberculosa dextra,
1 × caries caput femoris dextra,
1 × caries ossis pubis.

V. Tuberculous ulcers were found :
all along the intestines : 31 ×
In the small intestine *only* : 10 ×
In the colon *only* : 12 ×

VI. In 24 women with bilateral lung cavities we found :
10 × tuberculous ulcers all along the intestines,
4 × tuberculous ulcers in the colon *only*,
4 × tuberculous ulcers in the small intestines only,
6 × no ulcers.

VII. In 24 cases with bilateral tuberculosis of the lungs without cavities, we found :
10 × tuberculous ulcers all along the intestines,
1 × tuberculous ulcers in the colon *only*,
2 × tuberculous ulcers in the small intestine *only*,
11 × no ulcers.

VIII. In 10 patients with cavities in the right lung combined with tuberculosis of the left lung (without cavities) we found:
7 × no ulcers in the intestines,
1 × ulcers all along the intestines,
2 × ulcers in the small intestine *only*.

IX. In 12 patients with cavities of the left lung combined with tuberculosis of the right lung (without cavities) we found:
6 × ulcers all along the intestines,
2 × ulcers in the colon *only*,
4 × no ulcers.

X. Summary of the lungs:

Cavities on the right + left	: 24 ×
Bilateral tuberculosis (without cavities)	: 24 ×
Cavities on the right + tuberculosis on the left	: 10 ×
Cavities on the left + tuberculosis on the right	: 12 ×
Cavities on the right + "free" left lungs	: 2 ×
Cavities on the left + "free" right lungs	: 0 ×
Both lungs "free"	: 3 ×
Left "free", tuberculosis on the right	: 6 × (without cavities)
Right "free", tuberculosis on the left	: 4 × (without cavities)

XI. Tuberculous ulcers of the larynx etc. we found 10 ×
5 × with bilateral lung-cavities,
4 × with lung-cavities of the right,
1 × with lung-cavities of the left.

XII. We also found:

Tuberculosis of the uterus	: 3 ×
Tuberculosis of the tubae	: 3 ×
Tuberculosis of the ovaria	: 5 ×
Tuberculosis of the vagina	: 1 ×
Tuberculosis of the suprarenal body	: 2 ×
Tuberculosis of the knee	: 2 ×
Tuberculosis of the right elbow	: 1 ×
Tuberculosis of the thoracal spine	: 2 ×
Coxitis tuberculosa	: 1 ×
Tuberculosis ossis pubis	: 1 ×
Tuberculosis of the ribs	: 1 ×

Summary of 208 Chinese men.

I. Right lung with cavities : 129 ×; left lung : 117 ×
Right lung with tuberculosis without cavities : 68 ×; left lung : 76 ×
Right lung "free" from tuberculous lesions : 11 ×; left lung : 15 ×
Bilateral cavities : 89 ×
Bilateral tuberculosis of the lungs without cavities : 37 ×
Bilateral "free" lungs : 4 ×

II. Tuberculosis of the bronchial glands : 97 ×
Tuberculosis of the mesenterial glands : 66 ×
Tuberculous ulcers in the small intestine : 94 ×
Tuberculous ulcers in the colon : 86 ×
Tuberculous ulcers of the larynx etc. : 39 ×

III. Tuberculosis of the peritoneum : 19 ×
Tuberculosis of the liver : 20 ×
Tuberculosis of the spleen : 29 ×
Tuberculosis of the kidneys : 23 ×
Meningitis tuberculosa : 12 ×
Phneumothorax left lung : 5 ×
Phneumothorax right lung : 2 ×

IV. With 4 pairs of "free" lungs we found :
2 with tuberculosis of the lumbar spine,
1 with tuberculosis of the bronchial glands *only*,
1 with tuberculosis of the bronchial glands with tuberculous ulcers of the small intestine.

V. Tuberculous ulcers all along the intestines : 66 ×
Tuberculous ulcers in the small intestine *only* : 28 ×
Tuberculous ulcers in the colon *only* : 20 ×

VI. In 89 cases of bilateral lung cavities we found :
35 × tuberculous ulcers all along the intestines,
16 × tuberculous ulcers in the colon *only*,
11 × tuberculous ulcers in the small intestine *only*,
27 × no ulcers.

VII. In 37 patients with bilateral tuberculosis of the lungs we found :
10 × tuberculous ulcers all along the intestines,
1 × tuberculous ulcers in the colon *only*,
10 × tuberculous ulcers in the small intestine *only*,
16 × no ulcers.

VIII. In 40 cases with right lung cavities only we found :
14 × tuberculous ulcers all along the intestines,
1 × tuberculous ulcers in the colon *only*,
1 × tuberculous ulcers in the small intestine *only*,
24 × no ulcers.

IX. In 28 cases with cavities of the left lung only, we found :
7 × tuberculous ulcers all along the intestines,
2 × tuberculous ulcers in the colon *only* ,
3 × tuberculous ulcers in the small intestine *only*,
16 × no ulcers.

X. Together with the 4 pairs of "free" lungs, no ulcers of the intestines were found.

XI. Summary of the lungs :

Bilateral cavities	: 89 ×		
Bilateral tuberculosis without cavities	: 37 ×		
Bilateral "free" lungs	: 4 ×		
Cavities of the right lung with tubercul. on the left	: 35 ×	Cavities of the left lung with "free" right lung	: 25 ×
Cavities of the right lung with "free" left lung	: 5 ×	Cavities of the left lung with "free" right lung	: 2 ×
Tuberculosis of the right lung with "free" left lung	: 6 ×	Tubercul. of the left lung with "free" right lung	: 4 ×

XII. The 39 cases of tuberculosis of the larynx were combined with :
21 × bilateral lung cavities,
10 × cavities of the right lung,
5 × cavities of the left lung,
3 × bilateral tuberculosis without cavities.

XIII. We also found :

Tuberculosis of the bladder	: 2 ×
Tuberculosis of the urethra	: 1 ×
Tuberculosis of the prostatus	: 7 ×
Tuberculosis of the epididymis	: 1 ×
Tuberculosis of the suprarenal body	: 3 ×
Tuberculosis of the pancreas	: 2 ×
Tuberculosis of the sternum	: 1 ×
Coxitis tuberculosa on one side	: 1 ×
Tuberculosis of *one* knee	: 2 ×
Tuberculosis of *one* foot	: 1 ×
Tuberculosis of the lumbar spine	: 4 ×

TABLE I.

Lungs	Chinese (208)	Javanese men (295)	Javanese women (85)	Index I
Bilateral lung cavities	42.5 %	40 %	28 %	% of the total amount of lungs from patients suffering from tuberculosis.
Bilateral tuberculosis without cavities	16 „	10 „	28 „	
Cavities in right lung, left lung without cavities	17 „	16½ „	12 „	
Cavities in left lung, right lung without cavities	12 „	17½ „	14 „	
Cavities in right lung, left lung „free"	2½ „	4 „	2½ „	
Cavities in left lung, right lung „free"	1½ „	1.4 „	0 „	
Tuberculosis of left lung, right lung „free"	2 „	4 „	4½ „	
Tuberculosis of right lung, left lung „free"	6 „	3 „	7 „	
Both lungs free from tuberculosis	2 „	4 „	3½ „	

TABLE II

Glands, Larynx, Peritoneum Meningitis, Liver, Spleen, Kidneys etc.	Chinese	Javanese men	Javanese women	Index II
Tuberculosis of the bronchial glands	46½ %	57½ %	47 %	% of the total amount of patients suffering from tuberculosis
Tuberculosis of the mesenterial glands	31 „	29½ „	41 „	
Tuberculosis of bronchial + mesenterial glands	22 „	22 „	23 „	
Tuberculosis of the larynx	18½ „	18½ „	11½ „	
Tuberculosis of the peritoneum	7 „	9 „	8 „	
Tuberculosis of the liver	12 „	9½ „	13 „	
„ „ „ spleen	16 „	14 „	20 „	
„ „ „ kidneys	15 „	11 „	20 „	
Menigitis tuberculosa	6½ „	5½ „	6 „	
Pneumothorax (left)	1½ „	2 „	0 „	
Pneumothorax (right)	1 „	1 „	0 „	
Miliair tuberculosis	1½ „	2 „	3½ „	
Tuberculosis of the bones	3 „	4 „	7 „	

TABLE III.

Intestinal ulcers	Chinese	Javanese men	Javanese women	Index III
Tuberculous ulcers all along the intestines	31½ %	27 %	36½ %	% of the total amount of intestines from tuberculous corpses
Tuberculous ulcers in the colon *only*	9 „	7 „	14 „	
Tuberculous ulcers in the small intestine *only*	13½ „	18 „	12 „	
No ulcers	46 „	47½ „	37½ „	

TABLE IV

Ulcers in the small and the big intestine at the same time are found with:	Chinese	Javanese men	Javanese women	Index IV
Bilateral lung cavities	53½ %	55 %	32½ %	% of the total number of ulcers of the small and big intestine at the same time
Bilateral tuberculosis of the lungs without cavities	15 „	6 „	32½ „	
Right lung cavities, left no cavities	21 „	20 „	3 „	
Left lung cavities, right no cavities	10½ „	16 „	18½ „	
Both lungs free from tuberculosis		2 „		
Right cavities, left lung free			3 „	
Left cavities, right lung free				
Right tuberculosis without cavities, left lung free			3 „	
Left tuberculosis without cavities, right lung free		½ „	6½ „	

TABLE V.

Tuberculous bronchial glands were combined with	Chinese	Javanese men	Javanese women	Index V
Bilateral cavities of the lungs	51 × or 54 %	84 × or 49 %	13 × or 32½ %	% of the total amount of corpses with tuberculous bronchial glands
Bilateral tuberculosis of the lungs without cavities	13 × „ 12½ „	15 × „ 9 „	14 × „ 35 „	
Right lung cavities, left no cavities	18 × „ 12½ „	29 × „ 17 „	4 × „ 10 „	
Left lung cavities, right no cavities	9 × „ 9 „	29 × „ 17 „	4 × „ 10 „	
Right lung cavities, left lung "free"	1 × „ 1 „	1 × „ ½ „		
Left lung cavities, right lung "free"	1 × „ 1 „	1 × „ ½ „		
Bilateral "free" lungs	2 × „ 2 „	4 × „ 2½ „		
Right tuberculosis without cavities, left lung "free"	1 × „ 1 „	4 × „ 2½ „	2 × „ 5 „	
Left tuberculosis without cavities, right lung "free"	1 × „ 1 „	3 × „ 1½ „	3 × „ 7½ „	

TABLE VI

Tuberculosis of the mesenterial glands are found with	Chinese men			Javanese men			Javanese women		
Tuberculous ulcers all along the intestine	51 %	34 ×	51 %	57 %	46 ×	52 %	64 %	20 ×	55½ %
Ulcers in the colon *only*	25 „	4 ×	8 „	31 „	16 ×	18 „	16 „	2 ×	5½ „
Ulcers in the small intestine *only*	53½ „	15 ×	22½ „	52 „	11 ×	12½ „	58 „	7 ×	19 „
No intestinal ulcers	of the different kinds of ulcers to be found in the intestines	13 ×	20 „ of the glands	of the different kinds of ulcers to be found in the intestines	15 ×	17 „ of the glands	of the different kinds of ulcers to be found in the intestines	7 ×	19 „ of the glands

Palaeontology. — *A contribution to the knowledge of the fossil fauna and flora of Neede.* By C. H. OOSTINGH and F. FLORSCHÜTZ. (Communicated by Prof. L. RUTTEN.)

(Communicated at the meeting of January 28, 1928).

About two kilometres to the north-west of Neede (in the province of Gelderland) at approximately 52° 9′ N. 6° 35′ E. clay is dug for the manufacture of bricks and tiles on the western slope of the hill by the firm of the Wed. S. TEN BOKKEL HUININK.

This clay occurs between 20 and 30 metres above sea-level and is covered by about 7 metres of typically southern sand and gravel on the surface of which northern erratics are strewn. Borings proved that sand of a fluviatile character is underlying the clay.

The result of an examination of the remains of molluscs, mosses, fruits and seeds (the very abundant wood and rests of leaves are not considered in this article) follows.

A. Molluscs from the clay of Neede.

P. HUFFNAGEL has in 1911 mentioned the occurrence of *Paludina diluviana* [1]) while P. G. KRAUSE has also met at Neede with *Pal. diluviana* associated with a *Valvata*, closely allied to *V. naticina* Menke, and with opercula of *Bithynia tentaculata* (L.) [2]).

Examination of a larger number of Needian molluscs generally led to similar results as I found the following species:

1. *Gyraulus sp.*
2. *Ancylus lacustris* (Linné); in limited numbers.
3. *Bithynia tentaculata* (Linné); opercula.
4. *Viviparus viviparus (Linné)* (= *V. fasciatus* (Müller)) [3]).

The last is the most conspicuous species and very abundant. Most specimens are crushed in various directions; a few only are undamaged. The bulk has more or less thick shells and in this respect agrees with the form described as *Paludina diluviana* Kunth [4]).

An examination of a large quantity of specimens had already convinced me that this so called fossil form cannot be sharply separated from the recent *Viviparus viviparus* (= *fasciatus*). This opinion I found confirmed

[1]) Tijdschr. Nederl. Aardrijksk. Gen., 2nd series, XXVIII, pp. 66—72.

[2]) Z. d. deutschen geol. Ges., 1914, Monatsber., 2.

[3]) *Viviparus* Montfort, 1810, is the oldest available name for this genus, usually named *Paludina* or *Vivipara*. Vivipare Lamarck, 1809, is a vernacular name.

[4]) Z. d. deutschen geol. Ges., 1865, pl. 7, fig. 8 *a—d*.

by a recent paper of K. HUCKE [1]) in which this author proves that *Pal. diluviana (Viviparus diluvianus)* is a habitat-modification of *V. fasciatus* and identical with the still living form from rapidly flowing rivers.

5. *Valvata (Cincinna) piscinalis* (Müller).

Some of the specimens approach the lacustrine and fluviatile forma *antiqua* Sowerby.

6. *Valvata (Cincinna) naticina* Menke ; numerous specimens.

A comparison with recent specimens proved the Needian form undoubtedly to belong to this species. *V. naticina* nowadays lives in larger rivers east of the Oder over a large area between the Baltic and the northwestern part of the Pontic seas [2]).

7. *Helicidae spec.* ; a single damaged shell.

8. *Pisidium spec. spec.*

Besides these shells of *Cypris spec. (Ostracoda)*, a small fish-vertebra and a small molar tooth of a mammal were found.

It is a striking fact that the true riverdwellers e.g. *V. viviparus*, forma *diluviana*, and *Valvata naticina* dominate among the molluscs found. Consequently there is no ground for the opinion that the clay in which they occur is a lacustrine deposit. Evidently we must consider it as a fluviatile deposit, probably from a slowly flowing branch of a larger river.

Valvata naticina only is of importance for the determination of the geological age of the Needian clay.

In western Europe this species no longer lives, but as a fossil it occurs in the old-pleistocene for instance at Mosbach near Wiesbaden, at Mauer near Heidelberg and at Hangenbieten near Strassbourg ; besides in that of the hill of Oermten near Geldern. Consequently the Needian clay is probably of old-pleistocene age ; possibly it is old-interglacial or still older.

I do not think that the occurrence of *V. naticina* allows us to form an opinion about the climate of the time at which the Needian clay was deposited, as since pleistocene times the area of this species has been reduced both in an eastern and in a western direction.

Medan (Sumatra), Febr. 1927. C. H. O.

B. The fossil flora of the clay of Neede.

On the plant-remains buried in the clay of Neede I found the following publications :

a. HUFFNAGEL said in "Opmerkingen naar aanleiding van J. VAN BAREN's Morfologische bouw van het diluvium ten oosten van den IJsel" (Tijdschrift van het Koninklijk Nederlandsch Aardrijkskundig Genootschap, 2nd series, volume XXVIII, 1911) that the Needian clay also

[1]) Zeitschr. f. Geschiebeforschung, I, pp. 145—150, 1925.

[2]) LINDHOLM in Archiv f. Molluskenkunde, LIX, pp. 20—33, 1927.

contained a flora which was still being examinated and in which probably a Betula-species was represented.

b. JONGMANS stated in "Jaarverslag der Rijksopsporing van Delfstoffen over 1911": "From younger, pleistocene, layers in the neighbourhood of Neede the districtsgeologist HUFFNAGEL and I gathered clay with plant-remains. However the material obtained was not abundant and scarcely sufficient to prove the sediments to be not older than pleistocene".

c. VAN BAREN mentioned in de "De bodem van Nederland", p. 626, black clay with remains of the fir, broken mammoth-bones, shark-teeth and the freshwatermollusc *Paludina diluviana*.

Among the plant-remains could be recognized:

I. *Archegoniatae:*

Drepanocladus C. Müll. spec.
Homalia trichomanoides (Schreb.) Bryol. eur.
Neckera complanata (L.) Hüben.

II. *Anthophyta:* seeds or fruits from:

Acer campestre L.
Ajuga reptans L.
Alnus spec.
Carex spec.
Chenopodium spec.
Cornus sanguinea L.
Euryale spec.
Nuphar luteum Sm.
Oenanthe spec.
Pirus spec.
Potamogeton spec.
Prunus spinosa L.
Ranunculus sceleratus L.
Ranunculus spec.
Rhamnus cf. *Frangula* L.
Rumex maritima L.
Sambucus nigra L.
Solanum cf. *Dulcamara* L.
Sparganium spec. spec.
Urtica spec.
Viburnum Opulus L.
Vitis vinifera L.

The three recorded mosses still are common in our country: *Drepanocladus* as a bog-dweller whereas *Neckera complanata* and *Homalia trichomanoides* occur in forests on the soil and on trunks of trees.

With the exception of *Euryale, Vitis* and *Pirus* the *Anthophyta* found are not foreign to our recent flora. Conspicuous are the large masses of *Alnus*-cones and -seeds.

Untill 1907 *Euryale* was represented in the litterature only by the living species *Euryale ferox* Salisb. which grows in tropical and subtropical Asia from Bengal to Japan and even in the upper Ussuri-region at 45° 56′ N. with a mean January-temperature of —18° C. and a mean year-temperature of scarcely +4° C. (C. A. WEBER: *Euryale europaea* nov. sp. foss. Berichte der deutschen botanischen Gesellschaft, Band XXV, 1907).

In 1907 WEBER (loc. cit.) described a fossil seed from Russia as *Euryale europaea* whilst CL. and E. M. REID gave the same specific name to seeds from the clay of Tegelen (The fossil flora of Tegelen-sur-Meuse, near

Venloo, in the province of Limburg. Verhandelingen der Koninklijke Akademie van Wetenschappen te Amsterdam, Tweede Sectie, Deel XIII, N°. 6, 1907) but afterwards substituted it by *Euryale limburgensis*.

Eight years later these British authors described as new species : *Euryale nodulosa* and *Euryale lissa*, respectively from the clays of Reuver and of Brunssum (The pliocene floras of the Dutch-Prussian border. Mededeelingen van de Rijksopsporing van Delfstoffen, N°. 6, 1915).

Although damaged, the Needian seed shows some characteristics of an *Euryale*-seed but differs, as may be seen from a comparison of the figures with those in the cited papers of WEBER and of the REIDS, so much from the five recorded species that for the present I feel inclined to consider it as a new species for which I propose the name *Euryale Reidiorum*.

Vitis vinifera nowadays does not occur wild farther north than Mannheim ; moreover it is questionable whether at its farthest northern habitats the plant is really wild or escaped from cultivation.

The seed brought to the genus *Pirus* agrees precisely with the cultivated *Pirus communis* L. but it is considerably smaller.

Velp (G.), Holland, F. F.
January 1928.

An anatomical examination of *Staphylea*-seeds, after the dutch text of this article was already printed, threw doubt on the real nature of the fossil figured as N°. 2 and referred to the genus *Euryale*. In some characters it resembles *Staphylea*.

We will have to await the finding of less damaged specimens before it will be possible to express a definite opinion.

F. F.

EXPLANATION OF THE PLATES.

Plate I.

Fig. 1 *Vitis vinifera* L. fossil, Neede. 3/1.[1]).
Fig. 2 *Euryale Reidiorum* 3/1[1]).
Fig. 3 „ „ 24/1[1]).
Fig. 4 *Euryale Reidiorum*, testa-surface. 240/1.
Fig. 5 *Euryale ferox* Salisb., testa-surface. 240/1.
Fig. 6 *Euryale europaea* C. A. WEBER, testa-surface. 160/1[2]).
Fig. 7 *Euryale limburgensis* CL. et E. M. REID, testa-surface. 240/1.
Fig. 8 *Euryale lissa* CL. et E. M. REID, testa-surface. 240/1.
Fig. 9 *Euryale nodulosa* CL. et E. M. REID, testa-surface. 385/1.

Plate II.

Fig. 10 *Euryale Reidiorum* cross-selection of testa. 80/1.
Fig. 11 *Euryale ferox* Salisb., cross-selection of testa. 80/1[1]).
Fig. 12 *Euryale europaea* C. A. WEBER, cross-section of testa. 95/1[2]).
Fig. 13 *Euryale limburgensis* CL. et E. M. REID, cross-section of testa. 70/1[1]).
Fig. 14 *Euryale lissa* CL. et E. M. REID, cross-section of testa. 80/1.
Fig. 15 *Euryale nodulosa* CL. et E. M. REID, cross-section of testa. 80/1.

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[1]) Photo C. COOLHAAS.
[2]) Photo C. COOLHAAS after C. A. WEBER.

PLATE I.

Fig. 1.

Fig. 2.

Fig. 3.

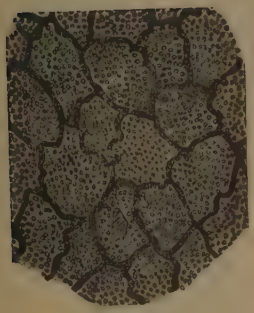


Fig. 4.

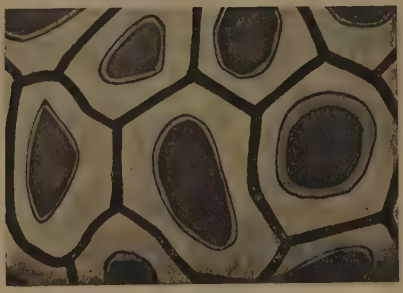
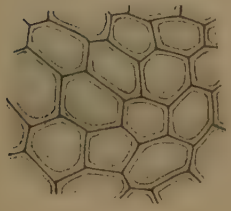


Fig. 5.



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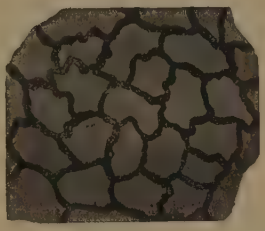

Fig. 7.

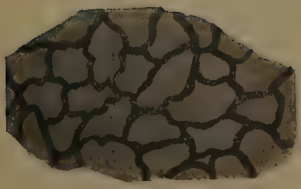

Fig. 8.

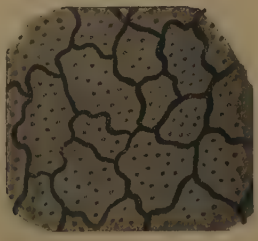

Fig. 9.

PLATE II.

Fig. 10.

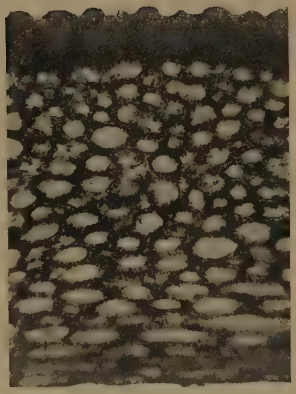

Fig. 11.

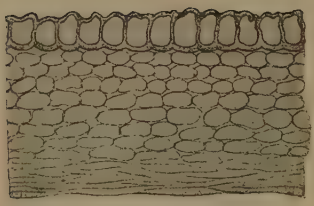

Fig. 12.

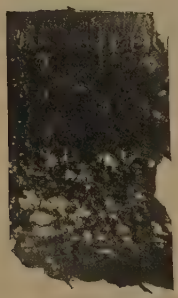

Fig. 13.

Fig. 14.

Fig. 15.

Mathematics. — *On non-holonomic connexions.* By Prof. J. A. SCHOUTEN (Communicated by Prof. JAN DE VRIES).

(Communicated at the meeting of February 25, 1928).

Introduction.

Non-holonomic parameters are well known in mechanics. In geometry they were first used by HESSENBERG [1]), by the author [2]), by CARTAN [3]) and by HLAVATY [4]). Using non-holonomic parameters, the equations of the general linear connexion get another form, given by HORAK [5]).

VRANCEANU [6]) has brought something essentially new. He has shown that in a V_n containing a non-V_m-building field of m-directions there exists a connexion for quantities belonging to the local R_m, and that this connexion can be deduced from the connexion of the V_n by the use of the coefficients of rotation of RICCI. HORAK [7]) has independently found this same connexion and in a paper that is to be published in a short time he will give especially mechanical applications.

Now we get a more general point of view starting from an A_n (X_n with a symmetrical linear connexion) containing a non-X_m-building field of m-directions. Then we can show that in the case of a general $(n\text{-}m)$-direction being given in every point (the case of „Einspannung" of WEYL) there is induced a connexion for all quantities belonging to the local E_m. So we get an A_n^m some of whose properties will be studied more in detail. Especially we will consider the properties of curvature and the generalised equations of GAUSS, which have the same form as in the case of an X_m in A_n, and also something will be said on the geodesics in A_n^m and A_n. Finally the "affine geometry" of an X_n^{n-1} in A_n will be treated. The first paragraph contains a short review on non-holonomic parameters in an A_n.

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[1]) Vektorielle Begründung der Differentialgeometrie, Math. Ann. **78** (18) 187—217.

[2]) Die direkte Analysis zur neueren Relativitätstheorie. Verh. Kon. Akad. v. Wet. Amsterdam **12** (18) 6.

[3]) Sur les variétés à connexion affine, Ann. de l'école normale (3) **40** (23) 325—412.

[4]) Sur le déplacement linéaire du point, Věstn. České Akademie (24) XIII 1—8.

[5]) Die Formeln für allgemeine lineare Uebertragung bei Benutzung von nichtholonomen Parametern, Nieuw Archief v. Wisk. **15** (27) 193—201.

[6]) Sur les espaces non holonomes, Comptes Rendus **183** (26) 825—854, Sur le calcul différentiel absolu pour les variétés non holonomes, Comptes Rendus **183** (26) 1083—1085.

[7]) (Czechisch) Sur une généralisation de la notion de variété, Publications de la Faculté des sciences de l'université Hasaryk, Brno.

§ 1. *Non-holonomic parameters in A_n.*

In every point of the A_n we introduce besides the measuring vectors $\overset{\nu}{e}_\lambda$, $\lambda, \mu, \nu = a_1, \ldots, a_n$, belonging to the variables x^ν, an arbitrary system $\overset{k}{e}_i$, $i, j, k = 1, \ldots, n$. Indicating the components with respect to the latter system with latin indices we have

$$v^k = v^\mu \overset{k}{e}_\mu \quad ; \quad w_i = w_\mu \overset{\mu}{e}_i \quad ; \quad \overset{k}{e}_i = \begin{cases} 1, i = k \\ 0, i \neq k \end{cases} \quad ; \quad \overset{k}{e}_i = \begin{cases} 1, i = k \\ 0, i \neq k \end{cases} . \quad (1)$$

$$A^\nu_\lambda = \begin{cases} 1, \lambda = \nu \\ 0, \lambda \neq \nu \end{cases} \quad ; \quad A^k_\lambda = A^\mu_\lambda \overset{k}{e}_\mu \quad ; \quad A^\nu_i = A^\nu_\mu \overset{\mu}{e}_i \quad ; \quad A^k_i = \begin{cases} 1, i = k \\ 0, i \neq k \end{cases} \quad (2)$$

and

$$v^k = A^k_\mu v^\mu \quad ; \quad w_i = A^\mu_i w_\mu . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

In the expression

$$(dx)^k = A^k_\mu \, dx^\mu \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

the x^k have a signification by themselves if and only if

$$\partial_{[\omega} A^k_{\mu]} = 0 \quad ; \quad \left(\partial_\omega = \frac{\partial}{\partial x^\omega} \right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

In the other case the x^k play the same part as the non-holonomic parameters in mechanics, only their differentials having a signification. We consider just this latter case. A symmetrical linear connexion being given by the parameters $\Gamma^\nu_{\lambda\mu}$ with respect to the system of measuring vectors $\begin{pmatrix} \nu \\ \lambda \end{pmatrix}$ we can fix this connexion also by parameters Λ^k_{ij} with respect to the system $\begin{pmatrix} k \\ i \end{pmatrix}$, so that

$$\begin{aligned} \nabla_j v^k &= A^{\beta k}_{j\gamma} \nabla_\beta v^\gamma = \partial_j v^k + \Lambda^k_{ij} v^i \\ \nabla_j w_i &= A^{\beta\alpha}_{ji} \nabla_\beta w_\alpha = \partial_j w_i - \Lambda^k_{ij} w_k \end{aligned} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

writing ∂_i for $A^\mu_i \partial_\mu$. It is easily found that

$$\Lambda^k_{ij} = \Gamma^k_{ij} + A^k_\alpha \partial_j A^\alpha_i = \Gamma^k_{ij} - A^\alpha_i \partial_j A^k_\alpha \quad . \quad . \quad . \quad . \quad . \quad (7)$$

hence

$$\Lambda^k_{[ij]} = - A^{\alpha\beta}_{ij} \partial_{[\beta} A^k_{\alpha]} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

So $\Lambda^k_{[ij]} = 0$ is necessary and sufficient for the holonomity of the x^k, hence, notwithstanding the symmetry in λ and μ of the $\Gamma^\nu_{\lambda\mu}$, the parameters Λ^k_{ij} with respect to non-holonomic parameters are not symmetrical in i and j. Of course $\Lambda^k_{[ij]}$ is no affinor. Applicating (6) on $\overset{k}{e}_i$ and $\overset{k}{e}_i$ we get

$$\Lambda^k_{ij} = \nabla_j \overset{k}{e}_i = - \nabla_j \overset{k}{e}_i \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

If the A_n passes into a V_n, and if we choose the orthogonal system $\overset{i}{i}{}^{\nu} = \overset{i}{i}{}^{\nu}, \overset{k}{i}_{\lambda}$, as measuring vectors $\overset{\nu}{\mathfrak{e}}, \overset{k}{\mathfrak{e}}_{\lambda}$, then

$$\Lambda^k_{ij} = -\nabla_j \overset{k}{i}_i = +\nabla_j \overset{i}{i}_k = \gamma_{kij} = -\gamma_{ikj} \quad . \quad . \quad . \quad (10)$$

where the γ_{kij} are the coefficients of rotation of RICCI. We will write $\gamma^k_{ij} = -\gamma^i_{kj}$ instead of $\gamma_{ikj} = -\gamma_{kij}$ to render more conspicuous the signification of these coefficients as parameters of a connexion. Hence in orthogonal components the connexion is given by

$$\nabla_j v^k = \nabla_j v_k = \partial_j v^k + \gamma^k_{ij} v^i = \partial_j v_k - \gamma^i_{kj} v_i \left(= \partial_j v_k + \sum_i \gamma_{ikj} v_i\right). \quad (11)$$

By differentiating and alternating (6) we get

$$\nabla_{[l} \nabla_{j]} v^k = \{\partial_{[l} \Lambda^k_{|h|j]} + \Lambda^k_{p\,[l} \Lambda^p_{|h|j]} - \Lambda^p_{[jl]} \Lambda^k_{hp}\} v^h \quad . \quad . \quad . \quad (12)$$

hence for the quantity of curvature follows

$$R_{ijh}{}^k = -2\,\partial_{[l} \Lambda^k_{|h|j]} - 2\,\Lambda^k_{p\,[l} \Lambda^p_{|h|j]} + 2\,\Lambda^p_{[jl]} \Lambda^k_{hp} \quad . \quad . \quad . \quad (13)$$

an equation passing into the ordinary form for $\Lambda^k_{[ij]} = o$, viz. for the case of holonomic parameters.

§ 2. *X^m_n in X_n.*

In every point of an X_n an m-direction be given. In the simplest case these m-directions are X_m-building, viz. it is possible to link them together in such a way that they build a system of ∞^{n-m} X_m. This simplest case will not be considered here. In general there will be a system of ∞^{n-M} X_M, $m < M \leqq n$, in such a manner that the given m-direction lies in every point in the local M-direction. We suppose that $M = n$ and give up all simplifications possible for $M < n$. An X_n equipped in this way with local m-directions will be called an X^m_n. An affinor of the local E_m, defined over X_n, will be called an "affinor of the X^m_n". Hence a field of contravariant vectors of the X^m_n is also a field of contravariant vectors of the X_n, but not vice versa, and a field of covariant vectors of the X_n (represented geometrically by two parallel E_{n-1} in every point) forms a field of covariant vectors of the X^m_n (represented geometrically by two parallel E_{m-1}) by intersection with the local E_m. All this is just the same as for an X_m in X_n.

We now introduce in every point of the X_n an $(n-m)$-direction, having no direction in common with the local m-direction. An X_n equipped in this way will be called *rigged* [1]). The essential difference between covariant vectors of the X_n and of the X^m_n disappears in a rigged X^m_n, because here every covariant vector of the X^m_n is in one to one correspondence with the covariant vector

[1]) WEYLS expression "eingespannt" being untranslatable and the $(n-m)$-direction reminding of a hoisted sail on a ship (the local m-direction), the word "rigged" was suggested.

of the X_n, whose E_{n-1} are determined by the E_{m-1} of the former vector and the $(n-m)$-direction of the rigging. Analytically we express this as follows. Besides the system $\begin{pmatrix} \nu \\ \lambda \end{pmatrix}$ belonging to the x^ν we introduce in every point a system $\begin{pmatrix} k \\ i \end{pmatrix}$, in such a manner that the first m contravariant measuring vectors e^ν, $a, b, c, d = 1, \ldots, m$, lie arbitrarily in the local m-direction, and that the other ones e^ν, $e, f, g, h = m+1, \ldots, n$, lie also arbitrarily in the local $(n-m)$-direction. With respect to this system the quantities of the X_n^m only have components with indices from 1 to m. Writing

$$B_\lambda^\nu = \overset{a}{e}_\lambda \overset{a}{e}^\nu \quad ; \quad C_\lambda^\nu = A_\lambda^\nu - B_\lambda^\nu = \overset{e}{e}_\lambda \overset{e}{e}^\nu \quad . \quad . \quad . \quad . \quad . \quad (14)$$

we have

$$B_a^c = \begin{cases} 1, a = c \\ 0, a \neq c \end{cases} \quad , \quad B_e^f = 0 \quad , \quad B_a^f = 0, \quad B_e^c = 0 \quad . \quad . \quad (15)$$

and the X_n^m-component of a vector of X_n is given by the equation

$$v'^\nu = B_\mu^\nu v^\mu \quad ; \quad w'_\lambda = B_\lambda^\mu w_\mu \, . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

or, with regard to the system $\begin{pmatrix} k \\ i \end{pmatrix}$:

$$v'^k = B_\mu^k v^\mu \quad ; \quad w'_l = B_l^\mu w_\mu \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

If we write $(dy)^c$ for the $\begin{pmatrix} k \\ i \end{pmatrix}$-components of a translation dx^ν lying in X_n^m:

$$dx^\nu = B_a^\nu (dy)^a \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

then the y^c are non-holonomic parameters and for B_a^ν follows

$$\boxed{B_a^\nu = \frac{\partial x^\nu}{(\partial y)^a}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

§ 3. *The connexion induced in a rigged X_n^m in A_n.*

By introducing the $\Gamma_{\lambda\mu}^\nu$ the X_n becomes an A_n. We are going to prove that the connexion of A_n induces a connexion in an X_n^m in A^n provided that this X_n^m is rigged. This connexion is defined as follows:

The covariant differential quotient of a quantity in X_n^m is the X_n^m-component of the covariant differential quotient in A_n.

Thus, indicating the covariant differential quotient in X_n^m by ∇' we have for vectors:

$$\left.\begin{aligned} \nabla'_\mu v^\nu &= B^{\beta\nu}_{\mu\gamma}\,\partial_\beta\, v^\gamma + B^{\beta\nu}_{\mu\gamma}\,\Gamma^\gamma_{\lambda\beta}\,v^\lambda \\ \nabla'_\mu w_\lambda &= B^{\beta\alpha}_{\mu\lambda}\,\partial_\beta w_\alpha - B^{\beta\alpha}_{\mu\lambda}\,\Gamma^\nu_{\alpha\beta}\,w_\nu \end{aligned}\right\} \quad \ldots\ldots \quad (20)$$

or, with regard to the system $\begin{pmatrix} k \\ i \end{pmatrix}$:

$$\left.\begin{aligned} \nabla'_b\, v^c &= B^{\mu c}_{b\nu}\,\nabla_\mu\, v^\nu = \partial_b\, v^c - B^\nu_a\, v^a\,\partial_b\, B^c_\nu + v^a\, B^{\mu c\lambda}_{b\nu a}\,\Gamma^\nu_{\lambda\mu} \\ \nabla'_b\, w_a &= B^{\mu\lambda}_{ba}\,\nabla_\mu\, w_\lambda = \partial_b\, w_a - w_c\, B^c_\lambda\,\partial_b\, B^\lambda_a - w_c\, B^{\mu\lambda c}_{ba\nu}\,\Gamma^\nu_{\lambda\mu} \end{aligned}\right\} \quad . \quad (21)$$

From this equation follows for the parameters Λ'^{c}_{ab} of the induced connexion

$$\boxed{\Lambda'^{c}_{ab} = B^{\lambda\mu c}_{ab\nu}\,\Gamma^\nu_{\lambda\mu} + B^c_\nu\,\partial_b\, B^\nu_a} \quad \ldots\ldots \quad (22)$$

Quite as in an A_n the alternating part

$$\Lambda'^{c}_{[ab]} = B^c_\nu\,\partial_{[b}\, B^\nu_{a]} \quad \ldots\ldots\ldots \quad (23)$$

is no affinor and depends on the choice of the systems $\begin{pmatrix} c \\ u \end{pmatrix}$.

A rigged X_n^m thus equipped with a connexion will be called an A_n^m. Applying (21) to $\underset{a}{e^c}$ and $\overset{c}{e}_a$ we get

$$\Lambda'^{c}_{ab} = \nabla'_b \underset{a}{e^c} = -\nabla'_b \overset{c}{e}_a. \quad \ldots\ldots\ldots \quad (24)$$

but also, in consequence of the choice of the measuring vectors

$$\Lambda'^{c}_{ab} = \nabla_b \underset{a}{e^c} = -\nabla_b \overset{c}{e}_a = \Lambda^c_{ab} \quad \ldots\ldots\ldots \quad (25)$$

Starting from a V_n instead of from an A_n and choosing for $\begin{pmatrix} k \\ i \end{pmatrix}$ an orthogonal system, we get easily

$$\Lambda'^{c}_{ab} = \gamma^c_{ab}\,(= -\gamma_{cab}) \quad \ldots\ldots\ldots \quad (26)$$

Hence the connexion induced in a V_n^m $(= X_n^m$ in $V_n)$ is obtained in a very simple manner by using the coefficients of rotation of RICCI with respect to a suitable chosen system of m congruences of curves [1]).

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[1]) VRANCEANU has found the connexion, induced in V_n^m, just in this way.

§ 4. *Properties of curvature of an* A_n^m *in* A_n.

We define the first and the second *affinor of curvature* in the same way as in an A_m in A_n:

$$\left.\begin{aligned} H_{ba}^{\cdot\cdot\nu} &= B_{ba}^{\mu\lambda}\,\nabla_\mu B_\lambda^\nu = -(\nabla_b' \overset{e}{e}_a)\,\overset{e}{e}^\nu = \Lambda_{ab}^{e}\,\overset{e}{e}^\nu \\ L_b^{\cdot c}{}_\lambda &= B_{bv}^{\mu c}\,\nabla_\mu B_\lambda^\nu = -(\nabla_b \overset{e}{e}^c)\,\overset{e}{e}_\lambda = -\Lambda_{eb}^{c}\,\overset{e}{e}_\lambda \end{aligned}\right\} \quad . \quad . \quad . \quad (27)$$

It strikes that, just as in an A_m in A_n, $H_{ba}^{\cdot\cdot\nu}$ lies with the index ν in the local $(n-m)$-direction and $L_b^{\cdot c}{}_\lambda$ contains with the index λ the local m-direction:

$$B_\alpha^\nu H_{ba}^{\cdot\cdot\alpha} = 0 \quad ; \quad B_\lambda^\alpha L_b^{\cdot c}{}_\alpha = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

But here $H_{ba}^{\cdot\cdot\nu}$ is no longer symmetrical in a and b, because

$$H_{[ba]}^{\cdot\cdot\nu} = -B_{ba}^{\mu\lambda}\,(\nabla_{[\mu}\overset{e}{e}_{\lambda]})\,\overset{e}{e}^\nu = \Lambda_{[ab]}^{e}\,\overset{e}{e}^\nu \quad . \quad . \quad . \quad . \quad . \quad (29)$$

and this expression vanishes if and only if all vectors $\overset{c}{e}_\lambda$ are X_{n-1}-building viz. if the field of m-directions is X_m-building, the case which we have excluded expressely.

It follows from

$$\left.\begin{aligned} H_{[ba]}^{\cdot\cdot\nu} &= B_{[ba]}^{\mu\lambda}\,(\partial_\mu B_\lambda^\nu + \Gamma_{\alpha\mu}^{\nu} B_\lambda^\alpha - \Gamma_{\lambda\mu}^{\alpha} B_\alpha^\nu) = B_{ba}^{\mu\lambda}\,\partial_{[\mu} B_{\lambda]}^\nu = \\ &= \partial_{[b} B_{a]}^\nu - B_\alpha^\nu\,\partial_{[b} B_{a]}^\alpha = C_\alpha^\nu\,\partial_{[b} B_{a]}^\alpha \end{aligned}\right\} \quad (30)$$

that the field of m-directions is X_m-building if and only if $C_\alpha^\nu\,\partial_{[b} B_{a]}^\alpha$ vanishes.

The ordinary method of obtaining the quantity of curvature is here useless because in an A_n^m *it is generally impossible to construct a parallelogram*, this impossibility being exactly characteristic for a non X_m-building field of m-directions. In fact, if on the one side a translation $\overset{1}{dy}^c$ is followed by a translation $\overset{2}{dy}^c$, and on the other side $\overset{2}{dy}^c$ by $\overset{1}{dy}^c$, then by using (23) and (30) we find for the closing vector the equation

$$2\,\overset{1}{d}y^b\,\overset{2}{d}y^a\,\partial_{[b} B_{a]}^\nu = 2\,\overset{1}{d}y^a\,\overset{2}{d}y^b\,(H_{[ba]}^{\cdot\cdot\nu} + \Lambda_{[ab]}^{c} B_c^\nu) \quad . \quad . \quad . \quad (31)$$

giving the decomposition into one component in the A_n^m and one in the local $(n-m)$-direction. The latter one only vanishes when the field of m-directions is X_m-building, the other one depends on the choice of the systems $\begin{pmatrix} c \\ a \end{pmatrix}$.

So we choose another way and start with $\nabla_{[l}' \nabla_{j]}' v^k$ which certainly is an affinor. We get

$$\left.\begin{aligned} &\nabla_{[d}' \nabla_{b]}' v^c = H_{[d\,b]}^{\cdot\cdot\alpha} B_\beta^c\,\nabla_\alpha v^\beta + \{H_{[d\,b]}^{\cdot\cdot\alpha}\,(B_a^\beta\,\partial_\alpha B_\beta^c - B_{\gamma a}^{c\beta}\,\Gamma_{\beta\alpha}^{\gamma}) + \\ &+ \partial_{[d}\,\Lambda_{|a|\,b]}^{c} + \Lambda_{p\,[d}^{c}\,\Lambda_{|a|\,b]}^{p} - \Lambda_{[b\,d]}^{p}\,\Lambda_{ap}^{c}\}\,v^a \quad ; \quad p = 1, \ldots, m. \end{aligned}\right\} \quad . \quad (32)$$

The expression corresponding with the right hand side of (13)[1]) is here no longer an affinor, but the expression

$$\left.\begin{aligned} R'^{\cdot\cdot\cdot c}_{d\,b\,a} = &-2\,H^{\cdot\cdot\alpha}_{[db]}\,(B^{\beta}_{a}\,\partial_{\alpha}\,B^{c}_{\beta} - B^{c\beta}_{\gamma a}\,\Gamma^{\gamma}_{\beta\alpha}) - 2\partial_{[d}\,\Lambda^{c}_{|a|\,b]} - 2\,\Lambda^{c}_{p\,[d}\,\Lambda^{p}_{|a|\,b]} + \\ &+ 2\,\Lambda^{p}_{[b\,d]}\,\Lambda^{c}_{ap} \quad ; \quad p = 1, \ldots, m, \end{aligned}\right\} \quad . \quad (33)$$

which we call the *quantity of curvature* of the A^m_n, is. Using the parameters belonging to the systems $\begin{pmatrix} k \\ i \end{pmatrix}$ and the equations (25) and (29), we may write for the first term of $R'^{\cdot\cdot\cdot c}_{dba}$

$$-2\,H^{\cdot\cdot\alpha}_{[db]}\,B^{\beta}_{a}\,\nabla_{\alpha}\overset{c}{e}_{\beta} = -2\,H^{\cdot\cdot e}_{[bd]}\,\Lambda^{c}_{ae} = 2\,\Lambda^{e}_{[bd]}\,\Lambda^{c}_{ae} \quad . \quad . \quad (34)$$

and this expression can be added to the last term of (33) so that finally

$$\left.\begin{aligned} R'^{\cdot\cdot\cdot c}_{dba} = &-2\partial_{[d}\,\Lambda^{c}_{|a|\,b]} - 2\,\Lambda^{c}_{p\,[d}\,\Lambda^{p}_{|a|\,b]} + 2\,\Lambda^{j}_{[b\,d]}\,\Lambda^{c}_{aj} \\ &p = 1, \ldots, m, \\ &j = 1, \ldots, n. \end{aligned}\right\} \quad . \quad . \quad (35)$$

From (32) and (33) follows

$$\nabla'_{[d}\nabla'_{b]}\,v^{c} = H^{\cdot\cdot\beta}_{[db]}\,B^{c}_{\gamma}\,\nabla_{\beta}\,v^{\gamma} - {}^{1}/_{2}\,R'^{\cdot\cdot\cdot c}_{dba}\,v^{a} \quad . \quad . \quad . \quad . \quad (36)$$

If the field of m-directions is X_m-building, then $H^{\cdot\cdot\gamma}_{[ba]}$ vanishes, (36) takes the ordinary form and (33) regains the same form as (13).

If the A_n passes into a V_n, the quantity of curvature passes into

$$\left.\begin{aligned} K'^{\cdot\cdot\cdot c}_{dba} = &-2\partial_{[d}\,\gamma^{c}_{|a|b]} - 2\,\gamma^{c}_{p\,[d}\,\gamma^{p}_{|a|b]} + 2\,\gamma^{j}_{[bd]}\,\gamma^{c}_{aj} \\ &p = 1, \ldots, m, \\ &j = 1, \ldots, n. \end{aligned}\right\} \quad . \quad (37)$$

§ 5. *The generalised equation of* GAUSS.

From the definition of the induced connexion it is easily deduced for a field v^c of the A^m_n:

$$\nabla'_{[d}\nabla'_{b]}\,v^{c} = H^{\cdot\cdot\beta}_{[db]}\,B^{c}_{\gamma}\,\nabla_{\beta}\,v^{\gamma} + L_{[d}{}^{c}{}_{|\alpha|}\,H^{\cdot\cdot\alpha}_{b]a}\,v^{a} - {}^{1}/_{2}\,B^{\partial\beta c}_{db\gamma}\,R_{\partial\beta\alpha}{}^{\gamma}\,v^{\alpha} \quad . \quad (38)$$

from which follows

$$\boxed{B^{\partial\beta\alpha c}_{dba\gamma}\,R_{\partial\beta\alpha}{}^{\cdot\cdot\gamma} = R'^{\cdot\cdot\cdot c}_{dba} + 2\,H^{\cdot\cdot\alpha}_{[b|a|}\,L_{d]}{}^{c}{}_{\alpha}} \quad . \quad . \quad . \quad : \quad (39)$$

This is the generalised equation of GAUSS for an A^m_n in A_n and we see that it has the same form as the equation for an A_m in A^*_n.

§ 6. *Geodesics in A^m_n and in A_n.*

A geodesic in A^m_n is a curve, generated by the pseudoparallel

[1]) This expression has been found by VRANCEANU but the affinor $K'^{\cdot\cdot\cdot c}_{dba}$ from (37) does not occur in his papers.

displacement of a contravariant vector in its own direction. t being a parameter on a geodesic, $\frac{dy^b}{dt} \nabla'_b \frac{dy^c}{dt}$ must have the direction of dy^c :

$$\frac{d^2y^c}{dt^2} + \Lambda_{ab}^{\ c} \frac{dy^a}{dt}\frac{dy^b}{dt} = \alpha \frac{dy^c}{dt} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (40)$$

Hence a geodesic in A_n^m is also a geodesic in A_n if and only if the vector

$$\frac{dx^\mu}{dt} \nabla_\mu \frac{dx^\nu}{dt} - \frac{dx^\mu}{dt} \nabla'_\mu \frac{dx^\nu}{dt} = \frac{dx^\mu}{dt} \left(\nabla_\mu \frac{dx^\alpha}{dt} \right) C_\alpha^\nu = \\ = \frac{dx^\mu}{dt} \frac{dx^\lambda}{dt} \nabla_\mu B_\lambda^\nu = \frac{dy^a}{dt} \frac{dy^b}{dt} H_{ba}^{\cdot\cdot\ \nu} \quad . \quad . \quad . \quad (41)$$

has the direction of dx^ν . In consequence the geodesics in A_n^m are always geodesics in A_n if and only if $H_{(ba)}^{\cdot\cdot\ \nu}$ vanishes. Thus the alternating part $H_{[ba]}^{\cdot\cdot\ \nu}$ which is of such a fundamental importance for the non-holonomity of A_n^m, has nothing to do with this question concerning the geodesics. To the case of a geodesic A_m in A_n corresponds the case of an A_n^m with $H_{(ba)}^{\cdot\cdot\ \nu} = 0$, all geodesics being also geodesics of A_n. If the A_n passes into a V_n , there exist also shortest curves in V_n^m. But it is immediately clear that shortest curves and geodesics are not identical here. In fact, through a point of V_n^m only ∞^{m-1} geodesics pass but generally ∞^{n-1} shortest curves, because every point of the V_n can be connected with every other point by a curve lying wholly in V_n^m. As an example we take the linear complex in R_3 belonging to a system of forces. The field of the 2-directions belonging to every point is not V_2-building and may be given by the equation

$$p_\lambda = a_\lambda + r^\varkappa f_{\varkappa\lambda} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (42)$$

a_λ being a constant vector, $f_{\lambda\mu}$ a constant bivector and r^ν the radius-vector. Writing p for the length of p_λ and i_λ for the unit vector belonging to p_λ we have

$$B_{\mu\lambda}^{\beta\alpha} \nabla_\beta i_\alpha = \frac{1}{p} B_{\mu\lambda}^{\beta\alpha} \nabla_\beta p_\alpha = \frac{1}{p} B_{\mu\lambda}^{\beta\alpha} f_{\beta\alpha} = \frac{1}{p} f'_{\mu\lambda} \quad . \quad . \quad . \quad (43)$$

and

$$H_{\mu\lambda}^{\cdot\cdot\ \nu} = -\frac{1}{p} f'_{\mu\lambda}\ i^\nu \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (44)$$

$f'_{\mu\lambda}$ being the V_3^2-component of $f_{\mu\lambda}$. The straight lines of the complex are geodesics as well in R_3 as in V_3^2. Obviously two arbitrary points in R_3 can not be connected by a geodesic of V_3^2 but always by a curve lying wholly in V_3^2. The quantity of curvature of V_3^2 is

$$K'_{db\,ac} = -2 H_{[b|a|}^{\cdot\cdot\ e} H_{d]ce} = -\frac{2}{p^2} f'_{[b|a|} f'_{a]c} \quad . \quad . \quad . \quad . \quad (45)$$

§ 7. *Affine geometry of an* X_n^{n-1} *in* A_n.

We will prove that an X_n^{n-1} in A_n determines an affine-normal direction in the same way as an X_{n-1} in A_n does, if the following two conditions are fulfilled.

1. The connexion in A_n leaves invariant each volume. (In E_n this condition is always fulfilled).

2. t_λ being a covariant vector having in every point the $(n-1)$-direction of the X_n^{n-1}, the affinor $h_{ba} = B_{ba}^{\mu\lambda} \nabla_\mu t_\lambda$ has the rank $n-1$.

If the connexion A_n leaves invariant every volume, there exists a constant n-vectorfield $P_{\lambda_1 \dots \lambda_n}$. Every other constant n-vectorfield can be obtained by multiplying $P_{\lambda_1 \dots \lambda_n}$ with a *constant* scalar. Now if h_{ba} has the rank $n-1$, t_λ can be chosen in a unique way, so that

$$t_{[\mu_1} t_{[\lambda_1} k_{\mu_2 \lambda_2} \dots k_{\mu_n] \lambda_n]} = P_{\mu_1 \dots \mu_n} P_{\lambda_1 \dots \lambda_n} \quad . \quad . \quad . \quad . \quad (46)$$

If the constant n-vectorfield be changed, t_λ only takes a *constant* scalar factor. The affine-normal vector can now be defined by means of the equations

$$\begin{aligned} t_\mu \, n^\mu &= 1 \\ B_a^\mu \, (\nabla_\mu \, t_\lambda) \, n^\lambda &= 0 \end{aligned} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (47)$$

h_{ba} having the rank $n-1$, n^ν is determined but for a *constant* scalar factor. Thus the affine-normal direction is found.

By use of the direction of n^ν just found, the X_n^{n-1} can be rigged, and an affine geometry can be obtained, as indicated in the former paragraphs.

Instead of h_{ba} also $k_{ba} = h_{(ba)}$ or $f_{ba} = h_{[ba]}$ can be used to construct the affine-normal direction.

Geology. — *Concerning the Tertiary of Atcheen.* By Prof. K. MARTIN.

(Communicated at the meeting of March 31, 1928).

In the north of Atcheen young tertiary sediments occur, that have been surveyed by the Survey-office of the Department of mines. More than 6000 mollusca from these deposits were examined to the following effect :

The fossils belong to 347 different species, which sometimes enable us to recognize distinctly a mutation of the species. So, for instance, a marked mutation occurs of the miocene *Melongena gigas*. To designate aberrant forms of species, already known from younger strata, the addition *prior* is adopted. Mutations and priores are of great significance in establishing the age of sediments.

Of the named species from Atcheen only 117 from the tertiary of Java are known, and most of them (77) are found in this island in the pliocene formation, while 66 % are still living in the present time. Moreover, a number of fossils are lacking that characterize the miocene of Java. So the strata of Atcheen must be considered to belong to the pliocene. As yet a perfectly similar formation from the East-Indian Islands is not known, but it may be that in the pliocene of Atcheen sediments occur that correspond to the Sondé-strata of Java.

The Palaeontologic data are insufficient for a grouping of the tertiary of Atcheen into zones of different ages, for the faunae of the consecutive zones, separated by the geologists, reveal a very close affinity. This appears not only from a direct comparison of all fossils, but also from the procentic amount of recent species, known in the several strata. The fossils examined have been gathered in a complex of 6 zones, whose thickness can be estimated at $\pm$ 3000 m. The pliocene geosyncline must, therefore, have subsided very quickly in this region, while the sedimentation was very intense.

The general character of the fauna is purely indopacific. European species are absolutely absent, and FROST's record of the presence of European fish-otoliths is erroneous. It is a mistake due to otoliths being unfit for the determination of the species. The connection between the Indian and the European sea was entirely broken during the pliocene time in the Indian Archipelago, as well as in British-India, where strata occur corresponding to the younger tertiary of Java.

An extensive treatise on the subject will appear in "Wetenschappelijke Mededeelingen", issued by the Mining Department in the Dutch East-Indies.

Chemistry. — *Optically active α-arsoncarboxylic acids.* By H. J. BACKER and C. H. K. MULDER. (Communicated by Prof. F. M. JAEGER.)

(Communicated at the meeting of March 31, 1928).

The optically active sulfocarboxylic acids show remarkable changes in rotatory power on neutralisation.

For the purpose of comparison we have now prepared some corresponding α-arsoncarboxylic acids. These acids are obtained in excellent yield by the reaction of the potassium salts of α-bromocarboxylic acids with potassium arsenite :

$$Br . CHR . CO_2K + K_3AsO_3 = AsO_3K_2 . CHR . CO_2K + KBr.$$

Only the first two terms of the series, arsonacetic acid [1]) and α-arsonpropionic acid [2]), are known. We have prepared α-arsonbutyric and α-arsonvaleric acids :

$$AsO_3H_2 . CH(C_2H_5) . CO_2H \qquad AsO_3H_2 . CH(C_3H_7) . CO_2H.$$

α-Arsonbutyric acid crystallises from water in anhydrous prisms and sometimes in plates melting at 127°. α-Arsonvaleric acid, recrystallised from the same solvent, separates in prisms or plates of melting point 114°. These acids may be titrated as dibasic acids, when using phenolphthalein as indicator.

The α-arsonic acids, derived from propionic, butyric and valeric acids, have an asymmetric carbon atom and should thus be resolvable into optically active enantiomorphs.

In fact the resolution has been successful by means of the secondary quinine salts.

α-Arsonpropionic acid gives a diquinine salt with 6 molecules of water ; the corresponding salt of α-arsonbutyric acid crystallises with 5 molecules, that of α-arsonvaleric acid with 4 molecules of water. The solubility of these diquinine salts in water is slight, namely about 0.8, 0.7 and 0.6 % respectively at room temperature.

The quinine salts may be recrystallised from dilute alcohol. They separate in concentric conglomerates of long needles.

Careful decomposition by baryta gave the barium salts, from which hydrochloric acid liberated the active acids.

About six recrystallisations were necessary to obtain the quinine salts in the pure active form.

The rotation of the acids and their salts was measured for different

[1]) PALMER, J. Amer. Chem. Soc. **45**, 3023 (1923). RAMBERG, Svensk Kem. Tidskr. **36**, 119 (1924).

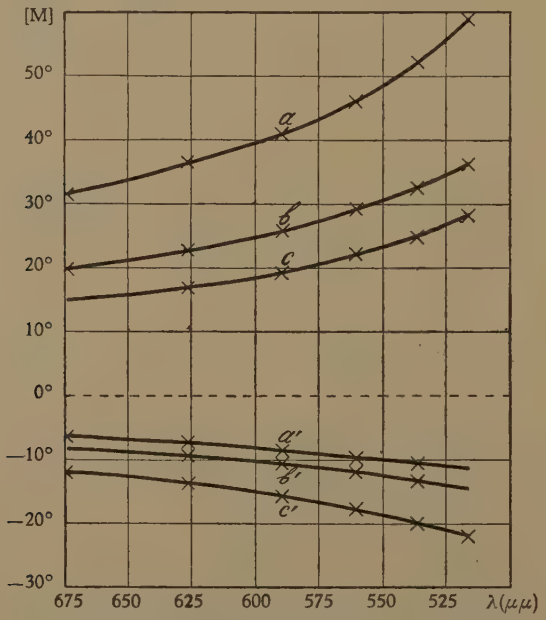
[2]) WIGREN, Svensk Kem. Tidskr. **36**, 127 (1924).

wave-lengths. The curves of rotatory dispersion have been plotted by means of the values, which are given in the following tables.

free acids	$\lambda(\mu\mu)$	674.0	626.5	589.3	560.5	536.5	516.0
d-α-arsonpropionic	[M]	+31°.5	+36°.1	+41°.0	+46°.0	+52°.2	+58°.9
d-α-arsonbutyric	„	+19.7	+22.7	+25.7	+29.1	+32.5	+36.3
d-α-arsonvaleric	„	—	+16.7	+19.3	+22.1	+24.8	+28.3

secondary barium salts	$\lambda(\mu\mu)$	674.0	626.5	589.3	560.5	536.5	516.0
d-α-arsonpropionic	[M]	− 6°.5	− 7°.3	− 8°.5	− 9°.7	−10°.5	—
d-α-arsonbutyric	„	—	− 9.4	−10.5	−11.9	−13.4	—
d-α-arsonvaleric	„	−10.2	−13.7	−15.6	−17.7	−20.0	−22.0

The tables and the figure show that neutralisation of two acid functions changes the sign of rotation.



Rotatory dispersion of α-arsoncarboxylic acids and their secondary barium salts.

a. α-Arsonpropionic acid *a′*. Salt.
b. α-Arsonbutyric acid. *b′*. Salt.
c. α-Arsonvaleric acid. *c′*. Salt.

The relation between the molecular rotation of the acid and the salt is —4.8 for *a*-arsonpropionic acid, —2.4 for *a*-arsonbutyric acid and —1.2 for *a*-arsonvaleric acid. Thus this relation changes gradually.

The study of the permanence of rotatory power has given remarkable results.

Whilst optically active sulfocarboxylic acids are less stable in alkaline than in acid solution, the arsoncarboxylic acids show just the opposite behaviour.

The barium salts of the active arsoncarboxylic acids are perfectly stable in aqueous solution, even at 100°.

The activity of the free acids, however, is labile, especially in the presence of strong inorganic acids.

The racemisation was examined kinetically; it proved to be a monomolecular reaction.

The constant increases almost proportionally to the quantity of hydrochloric acid added. The racemisation constant of the pure acids increases with their molecular weight.

A detailed account of the *a*-arsoncarboxylic acids and their derivatives will be published elsewhere.

Organic Chemical Laboratory of the State University.

Groningen.

Palæontology. — *The Law of the Necessary Phylogenetic Perfection of the Psychoencephalon.* By Prof. EUG. DUBOIS.

(Communicated at the meeting of March 31, 1928).

Clearly palæontology bears evidence of the growth of life on the Earth. Complex living beings, highly organized animals and plants have appeared after less complex living beings, simply organized animals and plants, taken on the whole in accordance with the natural systems. Of the Vertebrates — to mention only this group of animals — we find first Fishes, then also Amphibia and Reptiles, and finally Birds and Mammals, and in successive Ages the dominant types are Fishes, Reptiles, Mammals. In the vegetable kingdom appear and become dominant in successive Ages the Cryptogams, the Gymnosperms, the Angiosperms. We see life extend in the water, on the land, in the air. The diversity of the forms of the living beings and of their functions is becoming greater and greater and the efficiency more and more manifold as adaptation to the great diversity of surroundings.

Besides, in successive geological levels we find sequences of animal forms. Following one after another, each form is supplanted by a new one resembling the preceding one closely, but distinguished from it by intensification, transformation, or weakening of some characters.

All these and similar facts can hardly be explained in another way than by descent of the living beings from each other.

The absence of proof of c o n t i n u o u s d e v e l o p m e n t should, however, also be considered as an other important result of the palæontological research of the earth's crust. The minutest transitions between the members of the many parts of lines of descent with which we became acquainted, were so regularly absent, that this absence can no longer be attributed to "the imperfection of the geological record". Every member of a sequence is s t e p w i s e distinguished from the preceding and the following one. Again and again we find p i l l a r s of the expected bridges, never arches. Thus, for instance, in the series of the *Equidae*, starting with *Eohippus*, of the *Proboscidea*, starting with *Moeritherium*.

The members of these sequences are i n t e r m e d i a t e f o r m s, no "transition forms", no "links". Indeed, repeatedly palæontology has removed from the hypothetical stock line its at first much made of transition forms, referring them to side branches of the genealogical tree.

Strictly speaking gradual transition is a priori impossible, because every species living as independent creature, being adapted to particular circumstances of life, must be specialized in its peculiar way.

As it must, however, be assumed that every animal and every plant, in the past as well as at present, owe their existence to another animal and another plant, the present vegetable and animal world must necessarily be connected to those of all the earlier epochs by ties of consanguinity, i.e. by descent or phylogenesis.

That we again and again meet only with pieces of these ties, and that gradual transition between creatures living independently as different species is a priori impossible, leads to the conclusion that the real transitions, the missing pieces of the ties of relationship, the arches of the bridges connecting the species, took place **in the embryonal period of the individual life,** before the independent existence of the individual.

Of this palæontology could not furnish any documents, because they never existed. *Pithecanthropus, Mesohippus, Procamelus, Moeritherium* are pillars of these bridges, the arches of which — we must now assume — will never be found.

But the animals possess a complex of organs, the brain, which by the palæontological way, c a n teach us something about this transition from species to species accomplished in the embryonic state. This applies in particular to the cerebrum or prosencephalon of the Vertebrata and first of all of the Mammals, that part of the encephalon of the latter which is the direct and most highly specialized organ of the psychic functions of the animal, the definite p s y c h o e n c e p h a l o n.

In contrast with almost all the rest of the body, the cells of which multiply by division throughout life, the process of cell division is here entirely restricted to the embryonal period of life. The condition reached in the embryonal period is, therefore, preserved all through life, and the adult psychoencephalon still shows the embryonal cell division term in its volume, which latter, although still increasing subsequently to birth, no longer increases by cell division. This is the reason why the relative volume of this organ (which can be examined by means of natural and artificial endocranial casts also of extinct species of animals) increases in its phylogenesis in the regular proportion of the terms of a geometrical series with the ratio 2, for the cell division is a repeated division into two.

Here appears a direct relation between ontogenesis and phylogenesis.

Phylogenetically the senses of sight, smell, and hearing, placed at the head end of the animal body, "distance receptors", progress apparently concomitant with the cephalisation of the nervous system. With their larger psychoencephalon than that of the earlier forms, the phylogenetically later mammalian forms have also larger eyes, and larger olfactory receptors (we find the cribriform plate of the ethmoid bone extended, indicative of an increased number of olfactory nerve fibres). Compare the skulls of *Lama* with *Procamelus* (Fig. 1), *Martes* with *Mustela* (Fig. 2).

The concomitant perfection of the senses, accompanied with particular or general perfection of the rest of the sensorium, and the motorium, is, however, combined with d i v e r s e m o d e s of specialization of the

whole animal, ensuing from adaption through habit with negative selection ("adaptiogenesis"). Evidently the d e g r e e of every kind of this diverse

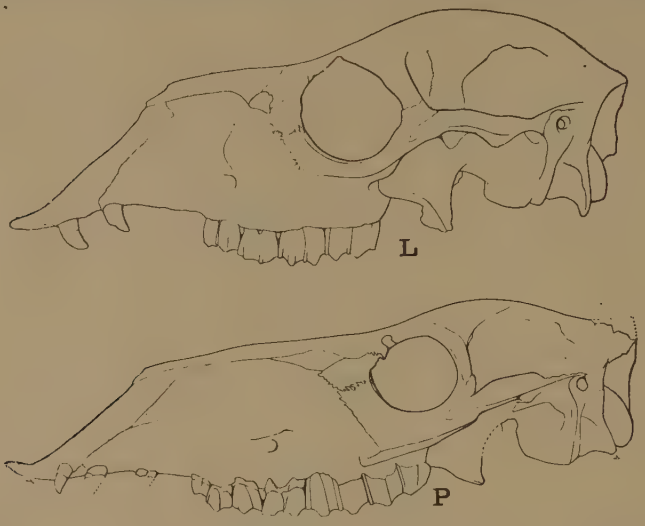


Fig. 1. *L.* Side view of skull of the present day *Lama huanachus* Molina. Accurate photographic outlines. 1/3 Nat. size.
P. Side view of skull of the Upper Miocene *Procamelus gracilis* Leidy. After COPE. 1/3 Nat. size. Bodily both Camelidae, the ancestral form and the descendant, are of the same size and general build.

specialization is determined by the degree of cephalization as given, in the Vertebrates, by the size of the encephalon and this in particular of the most central organ of the psychic functions, that part of the encephalon which in morphology is named the prosencephalon, and may according to its functions be rightly termed the psychoencephalon.

The increase of the size of the prosencephalon in the series of the Vertebrates, from the Fishes to the Mammals, has long been known. It is very striking that such a progress also exists between mammalian species of Early Tertiary and of recent times, belonging to the same group.

With regard to the volume of the whole encephalon in proportion to the volume of the body, this progress was first discovered by ED. LARTET, in 1868, by comparing some mammals (ungulates and carnivores) from the Lower Tertiary of France with recent species. CH. DARWIN made mention of this "remarkable conclusion", in which he saw evidence in support of gradual increase of the brain, in his "Descent of Man" (1871). LARTET thought that by way of explanation, he had to suppose a "tendance de la nature animée vers un perfectionnement dont la cause resterait toujours agissante et la limite indéfinie".

The fact of the small size of the brains of Lower Tertiary mammals was established by O. C. MARSH for a number of species in North-America

(1874—1897). Increase of brain volume was also shown by him for Birds (1880) and Reptiles (summarized 1896) of the Cretaceous period compared with the corresponding forms of recent times.

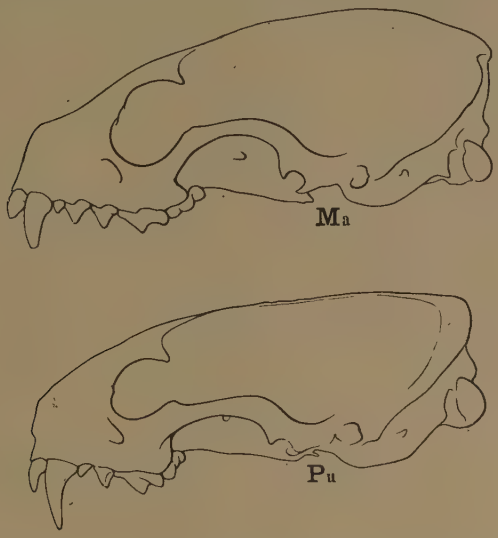


Fig. 2. *Ma.* Side view of skull of a full-grown *Martes foina* Erxl. ♀. Body weight 1160 g (somewhat below average of species).
Pu. Side view of skull of a full-grown *Mustela putorius* L. ♂. Body weight 1220 g (somewhat above average of species). Both accurate photographic outlines. Nat. size.

Concerning the extinct mammals throughout Tertiary time he drew up the following "general law of brain-growth" :

1. "All Tertiary mammals had small brains."

2. "There was a general increase in the size of the brain during this period."

3. "This increase was confined mainly to the cerebral hemispheres, or higher portion of the brain."

To this he added :

1. "The brain of a mammal belonging to a vigourous race, fitted for a long survival, is larger than the average brain, of that period, in the same group."

2. "The brain of a mammal of a declining race is smaller than the average of its contemporaries of the same group."

Accordingly, he quite adopted a causal nexus with the Spencerian "survival of the fittest", the Darwinian "natural selection".

Though it was known to him that "in any comparison of the size of the brain in different animals, whether in the same group or in others widely different, it is important to bear in mind that the brain of small animals is

proportionally larger in bulk than that of large animals", he was of opinion that this fact "can have only a limited effect, which would not change, essentially, the general results". Thus, as we know now, underrating the effect of the body size, MARSH was led to figure the brains enclosed in the skulls of small and large mammalian species at equal cranial length, through which disproportionally the former became too large, the latter too small.

It is mainly due to this underrating of the effect of the body size that the first two articles of the "general law of brain-growth", drawn up by MARSH, are not valid. Not all the Tertiary Mammals had small brains. Many from the Upper Tertiary are not inferior, as regards the size of the encephalon, to the recent species. Nor has the increase of the size of the brain during the Tertiary period been general. In the lowest cephalized Mammalia of the present day, many belonging to the Insectivora and the Chiroptera, such an increase has not taken place. Further we know now that it was not selection that has increased the volume of the encephalon.

Only the third article of the law of MARSH appears to be valid, but without the word "mainly", for the increase in size was exclusively confined to the cerebral hemispheres, the prosencephalon, in many later mammal species in comparison with earlier species.

Also in the present day animal world this phylogenetic brain-growth, in particular the growth of the prosencephalon, is to be recognized by comparison with allied forms, because some families and genera are more specialized, more progressive, particularly in this organ, than others.

Especially in the Vertebrata it was found in every class, but in the Mammalia in many cases, that the more specialized forms of a group, hence the progressive ones, regularly possess much greater relative brain volume than the conservative species, that are still more closely allied to the original forms.

The "comparative-anatomic" method of inquiry into the phylogenesis thus possible has very particular advantages here, as the size of the body can mostly only be determined accurately in living species.

For in every animal the volume of the brain is not only dependent on cephalization, but evidently also on the size of the body. The volume (and with this the weight) of the brain is really to be considered as the product of two factors. One corresponds to the degree of organization of the organ, the cephalization, the other depends on the bulk of the body. If the factor or coefficient of cephalization is called k, the brain weight E, the body weight P, E appears in general to be proportional to kP , when full-grown animals of two species are compared. Hence the coefficient of cephalization k is proportional to $\frac{E}{P^{5/9}}$. Between two full-grown individuals of one and the same species living in the natural state, the somatic factor is $P^{5/12}$. These are laws valid for all the classes of Vertebrates. For domesticated species (among others Man), the exponent is somewhat smaller.

With the phylogenetic exponent $^5/_9$ we now get not only equal values of

k for animal species which are considered as similar in their whole organization, for instance the members of the very homomorphous family of the *Felidae*, which differ so greatly in body size and weight (the latter varying in the ratio 1 to 100), but also a sequence of the Mammals which — leaving aside certain exceptions to the rule — is nowhere in contradiction to the natural system, if the k is calculated for the psychoencephalon.

If the value of the k of Man is put as unit, the coefficient in the Anthropoid Apes, *Hylobates* included, is $^1/_4$, in the *Canidae*, *Felidae*, and also the *Bovidae* and *Cervidae* differing little from $^1/_8$, in the *Leporidae* $^1/_{16}$, in the *Muridae* $^1/_{32}$, and in the *Soricidae*, *Centetidae*, and most *Microchiroptera* $^1/_{64}$. Small deviations in some groups are due, as has appeared, to unimportant secondary variations in the cerebellum or some parts of the prosencephalon.

In all these calculations the encephalon with its different, functionally inequivalent parts, was at first considered as a whole. According to what had been observed in the encephalon of Lower Tertiary mammals, strictly the psychoencephalon alone should have been taken into consideration. But between the living mammalian species, leaving the families of the *Insectivora* and *Microchiroptera*, which are on the lowest level of cephalization, out of account, the ratio of volume and weight of the psychoencephalon to the other parts of the brain, which moreover in most cases have a subordinate volume and weight, differ little. Hence the relations of the quantity found for the encephalon to those of the body can in general be considered as also valid for the psychoencephalon. As was already said before, the psychoencephalon has, however, a much smaller relative share of the total volume of the encephalon in the Lower Tertiary Mammals and also in many present day *Insectivora* and *Microchiroptera*. Besides, there are among the living mammalian species some in which the volume of the cerebellum has considerably increased secondarily and independently, namely doubled. In all these cases the cephalization of the psychoencephalon must be taken into consideration in itself, which has been done in the above sequence.

There is no need at present to point out that all the relations of quantity of the brain and the body are based on quantity relations of the cells, the neurones, to the body [1]). We may only remind here, that between the volume of ganglion cells and the body the same relation exists as between the weights of the brain and the body in the individuals of the same species.

This fact and direct counting prove that the number of the brain cells (neurones) is characteristic of the species, and remains the same for all individuals. Only the volume of the brain neurones increases, according to the above relation, with the body weight.

In homoneuric, but larger species, also the number of the brain neurones increases in the same proportion to the body weight.

With higher cephalization the number of the brain neurones increases,

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[1]) I may refer for this to my earlier communications.

independently of the body size, proportionally to the cephalization coefficient. If, therefore, two differently cephalized species of the same body size are compared, the number of nerve elements (neurones) in the brain varies as the value of the cephalization coefficient k, between two species of different body size as $kP^{5/9}$.

The existence of the relations of quantity of the central nervous system, and in particular of the psychoencephalon to the whole body proves that the integrative action of this organ also extends to the ontogenetic growth or growth outside the individual and the phylogenetic growth or growth outside the species.

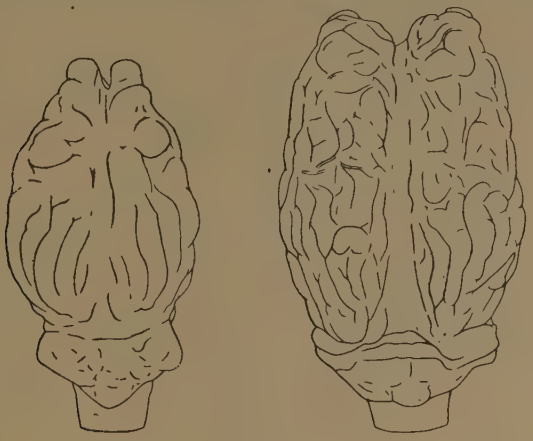
Now not a few species are seen to fall outside the sequences obtained by the method described, which taken on the whole, correspond to the natural zoological system, often outside their order or family, in some cases outside their genus, and this with a value of the cephalization coefficient k which is double or fourfold that of their nearest relations. A single recent form is seen falling outside the series with half the cephalization coefficient of its nearest relations, thus *Tragulus*. Striking is, for instance, the doubling of the coefficient in the genus *Martes* compared with the genus *Mustela* and the fourfold value in the *Megachiroptera* in comparison with most *Microchiroptera* [1]), also in *Tupaia* in comparison with *Sorex*. With regard to some extinct forms there are well-founded suspicions of phylogenetic relation with living species. When also the body size was known to some extent, I have included them in my comparison, thus *Pithecanthropus*, *Mesohippus*, *Procamelus*, *Moeritherium*, (?) *Proviverra (Cynohyaenodon)*. The affinity between *Palaeosyops* and *Tapirus* is only a less close one, but presumably an intermediate form connected these two Perissodactyles. Certainly the selenodont *Oreodon (Merycoidodon)* is still more distantly related with *Tayassus*, with which, however, it possesses some primitive characters in common.

It was really striking how regularly and accurately the determinations of the coefficients of cephalization led to equal results. The coefficient, calculated from the volume of the psychoencephalon, measured as exactly as possible, or from its weight, ascertained as exactly as possible, and the body weight of the species ascertained or estimated as accurately as possible [2]) appeared to increase exactly or very nearly by duplication,

[1]) In this case triplication was formerly found by comparison with the whole encephalon, in consequence of the fact that the cerebellum is very much larger relative to the prosencephalon in the *Microchiroptera* than in the *Megachiroptera*. For the prosencephalon quadruplication holds good.

[2]) In some cases, as in that of *Procamelus gracilis* Leidy and *Lama huanachus* Molina, to a certain extent also that of *Merycoidodon culbertsoni* Leidy and *Tayassus tajacu* L., the animals are of the same size and general build, so that the somatic factor may be eliminated in the equation. In most of the cases of fossil forms the body weight had to be estimated from comparison with similar living forms, however, an estimation of the body weight, even a roughly approximative one, may be serviceable, because it enters into the calculation with the low exponent $5/9$.

quadruplication, octuplication. A case of duplication between animals of the same size and general build is represented in Fig. 3. Further than the fourth term *(Moeritherium* to *Elephas)* the geometrical series is not continued in the following table, but undoubtedly the cephalization of some Lower Eocene



P L

Fig. 3. Superior view of endocranial cast of *Procamelus gracilis* Leidy (*P*) and *Lama huanachus* Molina (*L*).
Both accurate photographic outlines. 1/2 Nat. Size.
The casts belong individually to the skulls of Fig. 1.

Ungulata was still less than 1/8 of that of their far relatives of recent times. In so far as their body weight can be estimated by comparison, the Lower Eocene *Phenacodus* and *Coryphodon* possessed only 1/16 of the psychocephalization of the present day *Pecora*, but the Middle Eocene *Uintatherium* already 1/8 of this group, which is at the average level of the recent Mammals. Also *Moeritherium*, of about the same time, possessed 1/8 of the psychocephalization coefficient of *Elephas*, but this latter genus belongs to the most highly cephalized present day Mammals. In the same way the stock forms of *Rhinoceros* and of *Equus*, genera which, as regards cephalization, are above the present average level of the Ungulates, were, in their time, distinguished by a comparatively high cephalization. These facts give further support to the results recorded in the subjoined table. (See following page).

In each of the 17 sequences of this table, mammal forms on a lower level of cephalization precede related forms of a higher level, according to the geometrical progression of the cephalization coefficients. It is a matter of course, that in most of the sequences the mentioned species, genera, and higher groups are not to be considered as direct ancestors of one another. In consequence the lower forms are seldom of smaller size than those on a higher level of cephalization, and this easily may be conceived, since e.g. in such a homomorphous genus as *Felis*, the weights of the species diverge

Geometrical progression (ratio 2) of the psychocephalization

1. *Simiidae* — *Pithecanthropus* (Upper Pliocene) — *Homo*
2. *Callithrix* – *Saimiri* — { *Ateles* / *Cebus* }
3. *Lemur* — *Daubentonia*
4. *Mesohippus* (Middle Oligocene) — *Equus*
5. *Palaeosyops* (Middle Eocene) — ? — *Tapirus*
6. *Tragulus* — *Pecora*
7. *Procamelus* (Upper Miocene) — *Lama*
8. *Merycoidodon* (Middle Oligocene) — *Tayassus*
9. *Hippopotamus* — *Choeropsis*
10. *Moeritherium* (Upper Eocene) — [*Palaeomastodon* (Oligocene)] — [*Mastodon* (Miocene)] — *Elephas*[1])
11. *Mustela* — *Martes*
12. *Ursus* — *Helarctos*
13. *Proviverra* (Middle Eocene) — *Paradoxurus*
14. *Muridae* — { *Dipodinae* / *Leporidae* }
15. *Manis javanica* — { *Manis gigantea* / *Manis tetradactyla* }
16. *Microchiroptera* — *Vampirus* — *Megachiroptera*
17. { *Sorex* / *Centetes* } — { *Talpa* / *Myogale* / *Erinaceus* / *Potamogale* } — *Tupaia*

in the ratio 1 : 100, and this divergence is still greater between Mammals belonging to different genera and of more distant kinship. In this way the lower form *Hippopotamus* (of which, however, several small fossil species are known) may be much larger (indeed 10 times heavier) than the higher form *Choeropsis*. Thus, in each stage of cephalization, we find the forms grow to very different sizes. Moreover they diverge greatly in form and function by specialization, strikingly so in the second term of our series 17. As very important in this respect may be mentioned here the fact, that the psychocephalization of the much specialized and gigantic *Arsinoitherium* from the Lower Oligocene is equal to that of the small present day *Procavia*[2]) and most probably the Lower Oligocene *Megalohyrax* and other *Saghatheriidae*, from which fact we may infer that they all descended from a common stock of the next lower level.

Seemingly between the forms of equal cephalization, there may exist

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[1]) Although the cephalization of *Palaeomastodon* and that of *Mastodon* are not yet ascertained, they have been placed in this series of *Proboscidea* as intermediate (if not transitional) forms, well-known in other respects.

[2]) The estimated weight of *Arsinoitherium* is about 700 times the weight of *Procavia*.

gradual transition of specialization, as there is gradual transition in the sizes of the animals. On a nearer view we find this to be impossible, as specialization, depending on adaptation, is only divergence of the mode of organization, not progress of its degree. On the other hand, progress of the degree of organization, indeed of animality, i.e. real evolution of life exists from term to term of psychocephalization.

The discontinuance in the phylogenetic ties thus undeniably exists in respect of the psychocephalization. But this discontinuance is a regular one, resulting from the property of living bodies of growing by cell division, the multiplication of the units that compose them.

Here clearly the development is autonomous, determined by internal factors resulting from a fundamental property of the living beings. Also phylogenesis appears here as a mode of growth, like ontogenesis.

Furthermore, ontogenetic and phylogenetic growth of the animal evidently depending on the integrative action of the psychoencephalon, we may infer that no gradual transition can exist between species of different cephalization, only abrupt change in the form and structure, real metamorphosis. The nevertheless necessary connection, as we saw, must have taken place in the embryonal period of animal life. A smaller phylogenetic distance between species than with doubled psychocephalization and corresponding progress of the whole organization is, therefore, not possible. Quite inconsistent with this conclusion appears to be the phylogenetic gradual transition required by darwinism.

Between animals of equal cephalization gradual transition appears as a matter of fact, manifesting itself in the seemingly unlimited diversity in the specializations of the organisms, of the present day and the past. But this apparent gradual transition, evidently resulting from habit and negative selection, is adaptiogenesis, irregular as depending on the multifarious fluctuation of the environments, to which the mode, not the degree of organization of the living being responds. This modification of the organs, apparently rapid at progress of cephalization, which compellingly excited it, is no phylogenetic growth, which, being autonomous, has a definite direction.

Besides, in the case of the psychoencephalon, phylogenetic growth means at the same time perfection of the organ, continual phylogenetic progress of its functions, for the higher organization is here immediately attained by the increase of the number of cells, which at once leads to more multiple combination, greater functional complexity, direct enlargement of the animal's outer world.

Here is a law of evolution come forth out of the nature of the living being itself, not imposed by the surroundings. CHARLES DARWIN might still think: "There is no evidence of any law of necessary development", and HERBERT SPENCER: "We find progression to result, not from a special, inherent tendency of living bodies, but from a general average effect of their relations to surrounding agencies". From what we now know of the evolution of the

psychoencephalon, i.e. the evolution of animality, it appears that there actually does exist such a law of phylogenesis, and that with progression, with perfecting.

It is self-evident that this perfecting, this steady progression cannot have been caused by factors outside the animal, to which darwinism ascribed phylogenesis. External factors can only have effected d i v e r s i t y of the animal forms and functions, by adaptation. It is inconceivable that they should have acted continually in one definite direction, that of perfection.

Physics — *A formula expressing the deflection of the plumb-line in the gravity anomalies and some formulae for the gravity-field and the gravity-potential outside the geoid.* By F. A. VENING MEINESZ.

(Communicated at the meeting of January 28, 1928).

§ 1. In the middle of the 19th century STOKES succeeded in deducing a formula, which expresses the distance between the geoid and some chosen spheroid in the gravity anomalies of the whole earth. These gravity anomalies have to be computed by taking the difference of the observed gravity, reduced to the geoid by free air reduction (reduction of FAYE), and the normal gravity value, corresponding to the spheroid, which has been chosen ; that is to say this normal value represents the gravity-field, which would exist on an outside potential surface of some theoretical earth of the same mass as the actual earth, for which this potential surface coincides with the spheroid. This condition determines the normal gravity-field completely, as a well-known potential theorem indicates. To the formula of the normal gravity may however be added any constant or any spherical harmonic of the first degree without changing the result given by the formula of STOKES. This is an advantage, which we will secure also in §§ 2 and 3 for some other formulae, as it renders harmless any error in the constant term of the formula for the normal gravity, which is not impossible so long as gravity is unknown for the greater part of the earth's surface.

The chosen spheroid has to fulfill the following conditions : The volume as well as the centre of gravity must coincide with the volume and the centre of gravity of the actual earth. The radius of curvature has not to deviate more from the earth's radius than in the ratio of the first order of the flattening and we assume further for the spheroid as well as for the geoid that the angle between the normal and the radius towards the earth's centre is of that same order. We suppose lastly that the distance between both surfaces is of the second order of the flattening in proportion to the earth's radius.

With these restrictions the spheroid is wholly arbitrary ; it may be an ellipsoid, but this is not necessary and it is not even necessary that it is a body of revolution. It may further be emphasized that the formula of STOKES, corresponding to the fixed relation between gravity-field and shape of the geoid, is independent of the distribution of the masses of the earth, which are the causes of both ; it is therefore independent of any assumption about the mass distribution.

It is clear that the existence of this fixed relation between the gravity-field and the shape of the geoid implies also a fixed relation between the

gravity and the position of the normal on the geoid, that is to say between the gravity and the deflection of the plumb-line [1]). It is the object of this paper to deduce the corresponding equation, expressing the deflection of the plumb-line in the gravity-anomalies.

The easiest way to deduce such an equation is the differentiation of the distance between the geoid and the spheroid, given by the formula of STOKES, according to a direction tangent to the geoid; we find in this way the angle between the normal on the geoid and the normal on the spheroid, which may be considered as the deflection of the plumb-line with regard to the chosen spheroid. It is however necessary first to clear up two points concerning the equation of STOKES.

The first point is how far this formula gives also the local deviations of the geoid. This is questionable because of the fact, that in deducing it, a constant value R is substituted at a certain moment for the earth's radius. HELMERT for instance expresses doubt about it in "Die Theorien der höheren Geodäsie", and considers the equation as only valid for giving the general shape of the geoid. Would this be true, then the result for the deflection of the plumb-line got in this way, would be valueless; it is therefore necessary to prove, as we think it is possible to do, that this doubt is not founded. In this regard we may draw attention to a remarkable paper of POINCARÉ: "Les mesures de gravité et la géodésie", which is not generally known to geodesists and which appeared in the "Bulletin Astronomique" of 1901. POINCARÉ makes a study of the whole problem and, without knowing apparently about the work of STOKES, deduces the same formula and enlarges specially on the possibility of determining with this formula the local shape of the geoid with this equation. We will look into this question more closely in § 4.

The second point, which has to be examined, is the question concerning the effect of the masses outside the geoid. In deducing the formula of STOKES or one of the formulae of this paper, it is supposed that there are no masses outside the geoid, so that their validity is questionable so long as there are such masses. We will eliminate this difficulty by considering a regulated earth, for which these masses are removed. As the masses outside the geoid are fairly well known, the changing over from the actual earth to the regulated one presents no difficulty: we can compute the effect of these masses and find in this way the changes of the gravity anomaly and of the geoid, caused by the removal of these masses. The change of the gravity anomalies is about the same as the ordinary BOUGUER reduction. After having determined the shape of the regulated geoid by introducing these reduced anomalies in the formula of STOKES, the actual geoid can be derived by applying the difference between both geoids. It may be remarked — LAMBERT drew first attention to this point — that the change of the

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[1]) See also of M. MARCEL BRILLOUIN: „Champ de grav. extérieur et densités internes", Comptes Rendus de l'Acad. d. Sciences, t. 180, and t. 184.

geoid will also bring along a shifting of the centre of gravity, so that the centre of gravity of the spheroid, which has to coincide with that of the regulated earth, will not quite coincide with that of the actual earth.

In order to lessen the difference between the geoids of the actual earth and of the regulated earth, of which we determine the shape with the formula of STOKES, we may partly compensate the taking away of the masses by adding at the same time corresponding masses inside the geoid. We may do this by applying the condensation method of HELMERT, who adds in the same vertical at a depth of 21 km the same quantity of mass as has been removed. Or we may do it according to the inversion method of RUDZKI, who compensates the removal of an outside mass at a distance l from the centre of the earth, by adding a mass, which is R/l times the former one, at a distance R^2/l from the earth's centre, R being the mean earth's radius. RUDZKI obtains in this way, that the geoid remains unchanged, so that the formula of STOKES gives at once the actual geoid. Or lastly we may follow the isostatic method, adding the same quantity of mass, as has been taken away, and distributing it inside the geoid according to one of the accepted methods. The reduction of the gravity anomalies, corresponding to this last method, is the ordinary isostatic reduction, if we extend this mass-regulation also to the oceanic part of the earth's crust, filling up the oceans till they have normal density with masses removed from the crust below. For the purpose, which we have in view here, this extension to the oceans would not of course be necessary.

Of the three compensation methods the isostatic one gives the greatest shift of the geoid, because the distance between the removed masses and the added masses is greatest. Still it seems to me that this method is preferable to the other ones. The greater shift of the geoid is no serious drawback, as it can be computed, and the method has the advantage that the field of gravity anomalies becomes more regular by the isostatic reduction than by any other: it removes as well as possible the effect of the local mass-irregularities in the crust. This advantage is worth mentioning because it makes each anomaly value representative for a greater area of the earth's surface, so that a certain limited number of anomalies will give a better image of the geoid.

When we adopt one of these methods for removing the outside masses in order to be able to apply the formulae on the regulated earth, we must take care of the following question. As has been remarked in the beginning, we have to reduce the observed gravity by free air reduction to the geoid, that is to say that we have now to reduce to the regulated geoid. We have therefore to apply an extra free air reduction for the distance between both geoids. If we take for instance the isostatic method of regulation we have the following series of reductions to the observed gravity: First the free air reduction to the actual geoid and then the reductions belonging to the regulation of the earth, including first the ordinary isostatic reduction, representing the change of gravity caused by the transport of masses and

then the free air reduction from the original geoid to the regulated geoid. This last reduction is the same as the reduction of BOWIE.

Objection might be made that the masses outside the actual geoid have been removed and not the masses outside the regulated geoid. As a matter of fact we ought to remove the masses outside this last geoid. The difference may however be neglected if one of the compensation methods has been used, as in this case the distance between both geoids is small; it will in fact not exceed some twenty meters.

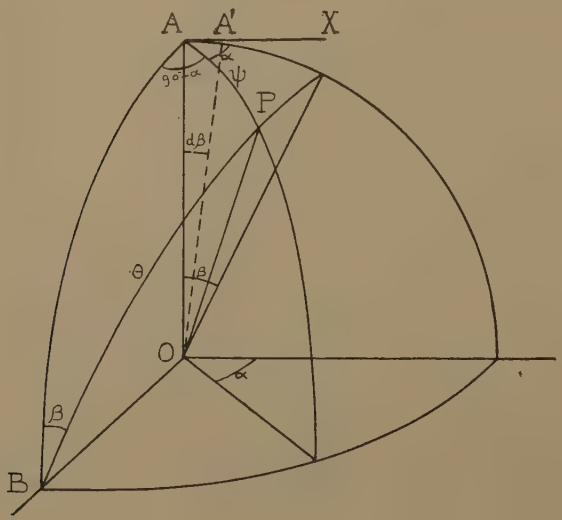
We will now proceed to the deduction of the formula for the deflection of the plumb-line, which we will do by differentiating the formula of STOKES according to a direction tangent to the geoid. This formula may be written:

$$N = \frac{R}{4\pi\gamma}\int^{\sigma} S_T \Delta_0 \, d\sigma \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1^A)$$

in which: N = distance between geoid and spheroid,
R = mean earth's radius,
$d\sigma$ = surface-element of a sphere with radius 1,
Δ_0 = gravity anomaly corresponding to the element $d\sigma$,
S_T = function of the angle ψ between the radius of the point A, where N is computed, and the radius of the point P coinciding with $d\sigma$:

$$S_T = \operatorname{cosec}\tfrac{1}{2}\psi + 1 - 6\sin\tfrac{1}{2}\psi - 5\cos\psi - 3\cos\psi \, lg\,[\sin\tfrac{1}{2}\psi\,(1+\sin\tfrac{1}{2}\psi)] \quad (1^B)$$

We choose in A an X direction tangent to the geoid and introduce an azimuthal coordinate α, representing the angle between OAX and OAP.



Besides the system of coordinates ψ and α we introduce a second system θ and β, θ being the angle between the radii of P and B (OB perpendicular to OAX), and β the angle between AOB and POB.

Let ϑ be the deflection of the plumb-line in A with regard to the spheroid, and let ϑ_x be its component in the X direction. Neglecting quantities of higher order, we have then:

$$\vartheta_x = \frac{\partial N}{\partial x} = \frac{1}{R}\frac{\partial N}{\partial \beta}$$

in which last formula N is expressed in the second system of coordinates θ and β:

$$N = \frac{R}{4\pi\gamma}\int^{\sigma} S_T(\theta,\beta) \times \triangle_0(\theta,\beta)\, d\sigma.$$

For differentiating N according to β we have to determine the value of $N + \frac{\partial N}{\partial \beta}\, d\beta$ in a point A' at a distance $Rd\beta$ from A. The field $\triangle_0$ remains at the same place, but the field S_T shifts with the centre A towards A', that is to say it rotates an angle $d\beta$. If the system of coordinates is kept unmoved, we find:

$$N + \frac{\partial N}{\partial \beta}\, d\beta = \frac{R}{4\pi\gamma}\int^{\sigma} S_T[\theta, (\beta - d\beta)] \times \triangle_0(\theta,\beta)\, d\sigma$$

but we may also shift the system of coordinates in the same way as the field S_T by rotating it an angle $d\beta$, and we find then:

$$N + \frac{\partial N}{\partial \beta}\, d\beta = \frac{R}{4\pi\gamma}\int^{\rho} S_T(\theta,\beta) \times \triangle_0[\theta, (\beta + d\beta)]\, d\sigma$$

The two expressions give the formulae:

$$\vartheta_x = \frac{1}{R}\frac{\partial N}{\partial \beta} = -\frac{1}{4\pi\gamma}\int^{\sigma} \frac{\partial S_T}{\partial \beta} \triangle_0\, d\sigma \quad . \quad . \quad . \quad . \quad . \quad (2^A)$$

$$\vartheta_x = \frac{1}{R}\frac{\partial N}{\partial \beta} = +\frac{1}{4\pi\gamma}\int^{\sigma} S_T \frac{\partial \triangle_0}{\partial \beta}\, d\sigma \quad . \quad . \quad . \quad . \quad . \quad (3^A)$$

which are both worth while examining: The first expresses ϑ_x in the gravity anomaly itself and the second expresses ϑ_x in the horizontal gradient of the anomaly.

We will go back to the original system of coordinates, and express both formulae in ψ and α. The spheric triangle APB gives:

$$\left.\begin{aligned} \frac{\partial \psi}{\partial \beta} &= \cos \alpha \\ \frac{\partial \alpha}{\partial \beta} &= -\sin \alpha \operatorname{cotg} \psi \end{aligned}\right\} \quad \ldots\ldots \quad (4^A)$$

and therefore:

$$\frac{\partial}{\partial \beta} = \cos \alpha \frac{\partial}{\partial \psi} - \sin \alpha \operatorname{cotg} \psi \frac{\partial}{\partial \alpha} \quad \ldots\ldots \quad (4^B)$$

so that we get:

$$\vartheta_x = -\frac{1}{4\pi\gamma} \int^{\sigma} \cos \alpha \frac{\partial S_T}{\partial \psi} \triangle_g \, d\sigma \quad \ldots\ldots \quad (2^B)$$

$$\vartheta_x = +\frac{1}{4\pi\gamma} \int^{\sigma} S_T \left[\cos \alpha \frac{\partial \triangle_0}{\partial \psi} - \sin \alpha \operatorname{cotg} \psi \frac{\partial \triangle_0}{\partial \alpha} \right] d\sigma \quad \ldots \quad (3^B)$$

We may introduce

$$d\sigma = \sin \psi \, d\alpha \, d\psi$$

and multiply with $\varrho'' = \operatorname{cosec} 1''$, in order to express ϑ_x in seconds. The second formula gives then:

$$\vartheta''_x = \frac{\varrho''}{4\pi\gamma} \int_0^{2\pi} \cos \alpha \, d\alpha \int_0^{\pi} \sin \psi \, S_T \frac{\partial \triangle_0}{\partial \psi} d\psi - \frac{\varrho''}{4\pi\gamma} \int_0^{2\pi} \sin \alpha \, d\alpha \int_0^{\pi} \cos \psi \, S_T \frac{\partial \triangle_0}{\partial \alpha} d\psi \quad (3^C)$$

which we will not further examine in this paper.

The first formula gives:

$$\vartheta''_x = \frac{1}{2\pi} \int_0^{2\pi} \cos \alpha \, d\alpha \int_0^{\pi} Q \triangle_0 \, d\psi \quad \ldots\ldots \quad (2^C)$$

with

$$Q = -\frac{\varrho''}{2\gamma} \sin \psi \frac{\partial S_T}{\partial \psi}$$

that is to say in introducing the expression for S_T:

$$\left.\begin{aligned} Q = \frac{\varrho''}{2\gamma} \cos^2 \tfrac{1}{2} \psi \Big[& \operatorname{cosec} \tfrac{1}{2} \psi + 12 \sin \tfrac{1}{2} \psi - 32 \sin^2 \tfrac{1}{2} \psi + \\ & + \frac{3}{1 + \sin \tfrac{1}{2} \psi} - 12 \sin^2 \tfrac{1}{2} \psi \, lg \left\{ \sin \tfrac{1}{2} \psi \, (1 + \sin \tfrac{1}{2} \psi) \right\} \Big] \end{aligned}\right\} \quad (2^D)$$

The following table gives the values of Q for intervals of 10° for ψ (Δ_0 and γ are supposed to be expressed in 0.001 cm):

ψ	Q	ψ	Q	ψ	Q
1°	+12.35	70°	+0.03	140°	−0.25
10°	+ 1.59	80°	−0.15	150°	−0.16
20°	+ 1.02	90°	−0.29	160°	−0.08
30°	+ 0.79	100°	−0.38	170°	−0.02
40°	+ 0.61	110°	−0.41	180°	−0.00
50°	+ 0.43	120°	−0.40		
60°	+ 0.22	130°	−0.34		

For increasing ψ, Q first decreases rather quickly, afterwards more slowly, is zero for ψ more than 70°, gets to a minimum −0.41 for ψ about 110°, and increases then towards zero, which value is reached for $\psi = 180°$.

For small ψ we can neglect in the formula for Q all but the first term, and we get by introducing in stead of ψ the linear distance $AP = r$, so that $r = 2R \sin \frac{1}{2} \psi$:

$$Q = \frac{1338}{r} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2^E)$$

in which r is expressed in km.

By substituting in (2C) the surface element df in stead of the expression in $d\psi$ and da, we find for the effect of the anomalies in the neighbouring region:

$$\vartheta''_x = \frac{\varrho''}{2\pi\gamma} \int \cos \alpha \frac{\Delta_0}{r^2} df \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

We may express this formula in the following way: If we apply to every surface element df of the geoid a mass equal to $\frac{\varrho''}{2\pi\gamma} \Delta_0 df$, the reflection of the plumb-line is the resultant of the attractions, exerted by these masses in A if these attractions act along the radii towards df and are equal to the mass divided by the square of the distance.

The decrease of the influence of a certain anomaly with the distance is therefore greater than for the formula of STOKES: This formula reduces for the neighbouring region to:

$$N = \frac{1}{2\pi\gamma} \int \frac{\Delta_0}{r} df \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

so that the influence on N decreases only in inverse ratio to the first power of r.

Formula (6) is in harmony with formula (5) as this last formula indeed represents the gradient of the former.

The property that the influence of a certain anomaly decreases more quickly with the distance than for the formula of STOKES, gives the advantage that the formula for the deflection of the plumb-line is more independent of the anomalies in other parts of the earth's surface, so that the fact, that as yet the gravity is only very imperfectly known over the globe, need not necessarily prevent its application.

The formula for the deflection of the plumb-line may be useful for connecting the results of both types of geodetic operations: the astronomic observations and the determinations of the gravity. Besides it may be of use, as well as the formula of STOKES itself, to study the local deviations of the geoid for the central area of a region where the gravity has been determined. The anomalies in the other parts of the globe will generally have an equable effect for the whole central area: for the deflection of the plumb-line for instance a small and nearly constant amount. So by neglecting the influence of the anomalies in those distant region we will get a result for the shape of the geoid, which will perhaps be slightly wrong in position, but which will not be much deformed. We may even put the question, if the determination of the geoid along this line will not better answer the purpose, than that, which is founded on the direct observations of the plumb-line deflection. The disadvantage of the uncertainty of the gravity in other parts of the globe, may perhaps be more than compensated by the uncertainty, resulting from the fact that in using this last method, N is determined by integrating a quantity, which is only known in isolated spots.

A provisional computation for the Netherlands has shown, that the differences of the plumb-line deflections for the three stations in the central part: Urk, Wolberg, and Utrecht, are in good harmony with the values which have been deduced from the gravity anomalies: the deviations do not exceed 0".5. For this computation only the gravity-anomalies in the Netherlands have been used.

It appears doubtful if the accuracy of the result can ever attain such a perfection, that the formula could be used to controll the results of the triangulations by giving for each astronomical station an equation for the latitude and longitude components of the plumb-line deflection. Still it may give a useful control for traverse surveys, of which the errors are so much greater, as is well known: We may considerably improve in this way the control of these surveys by astronomical observations. For this purpose it is of course necessary to have a sufficient number of gravity stations in an area of somewhat greater extension than the area of the survey.

Besides the formulae for the deflection of the plumb-line, it is a simple matter to deduce other formulae from the formula of STOKES by further differentiation. In this way we may derive formulae, expressing the second differential coefficients of N in the gravity anomalies or in the gradient of

these anomalies. These second differential coefficients may be considered as the difference of the reciprocal values of the radii of curvature of the geoid and the spheroid, so that the curvature of the geoid may be determined in this way. We will not take up this question in this paper and we will only remark that in this way a formula may be found, connecting the two series of quantities, given by the torsion balance : the horizontal gradients of the gravity and the curvature data. These data are therefore not independent, although the curvature cannot be brought in connection with a single value of the gradient but with the gradient field over the whole earth.

The next paragraphs will treat of the neglected terms of the formula of STOKES and will prove the validity of this formula and of the formulae found by differentiating it, as well for the determination of the general shape of the geoid as for that of the local irregularities. Besides they will treat of some formulae about the gravity field outside the geoid, which we will need for our purpose.

§ 2. *Gravity anomaly in an arbitrary point outside a sphere, which encloses all the masses and on which the anomaly is known.*

Before looking into the question of the validity of the formula of STOKES, we will derive the solution of the above problem, which will be wanted for that research. Practically this will also give the formula for expressing the gravity anomaly outside the geoid in the anomalies on the geoid : the error committed by replacing the geoid by the sphere with a radius equal to the mean earth-radius is of the same order as the error in the STOKES formula i.e. the percentual error will not exceed the order of the flattening.

Let T be the difference of the potentials caused in the same point by the actual earth and by the theoretical earth ; by the last we mean one of the infinite number of possible mass-distributions inside the geoid with the same total mass as the earth and with an outside potential surface coinciding with the spheroid. Only values of T in points outside both mass-distributions will be considered, that is to say in points outside the geoid or on the geoid.

Let further :

a = aequatorial radius of the spheroid,
f = flattening of the spheroid,
g = gravity in an arbitrary point, caused by the actual earth,
γ = gravity in an arbitrary point, caused by the theoretical earth,
ϱ = radius of an arbitrary point towards the centre of gravity of geoid and spheroid,
g_0, γ_0, ϱ_0 = values of g, γ and ϱ on the geoid,
g_s, γ_s, ϱ_s = values of g, γ and ϱ on the spheroid,
N_0 = distance between geoid and spheroid,
R = radius of the sphere, which is considered,
$\Delta_0 = g_0 - \gamma_s$ = gravity anomaly on the geoid.

As we suppose N_0 to be of the order of (af^2), Δ_0 of the order of (gf^2) and the angle between ϱ and g or γ of the order of (f), we have:

$$N_0 = \frac{T_0}{\gamma_0} + (af^4) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7^A)$$

$$g_0 - \gamma_0 = -\frac{\partial T_0}{\partial \varrho} + (gf^4) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7^B)$$

$$\gamma_0 - \gamma_s = N_0 \frac{\partial \gamma_0}{\partial \varrho} + (gf^4) = \frac{\partial \gamma_0}{\partial \varrho} \frac{T_0}{\gamma_0} + (gf^4) \quad . \quad . \quad . \quad . \quad (7^C)$$

Therefore:

$$\Delta_0 = -\frac{\partial T_0}{\partial \varrho} + \frac{\partial \gamma_0}{\partial \varrho} \frac{T_0}{\gamma_0} + (gf^4)\ ^1), \quad . \quad . \quad . \quad . \quad . \quad . \quad (7^D)$$

In the same way as the anomaly Δ_0 has been defined for the geoid, we may define the anomaly Δ for a point outside the geoid as the difference of the actual gravity in this point and the gravity of the theoretical earth in a point in the same vertical, where the potential is equal to the potential of the actual earth in the first point. By rough approximation we may say that the second point is at the same distance outside the spheroid as the first point is outside the geoid. We have again:

$$\Delta = -\frac{\partial T}{\partial \varrho} + \frac{\partial \gamma}{\partial \varrho} \frac{T}{\gamma} + (gf^4) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7^E)$$

If the spheroid were a sphere, the differential-quotient $\frac{\partial \gamma}{\partial \varrho}$ would be:

$$\frac{\partial \gamma}{\partial \varrho} = -2\frac{\gamma}{\varrho} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

For a spheroid it has to be multiplied by a factor $1 + (f)$ (see e.g. HELMERT, Theor. d. h. Geod. II, page 94) and so we get:

$$\Delta = -\frac{\partial T}{\partial \varrho} - 2\frac{T}{\varrho} + (gf^3) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

with a corresponding equation for Δ_0. As Δ is of the order of (gf^2) the last term in (9) is small with regard to the others. It will in fact not surpass 0.0001 or 0.0002 cm in Δ. We will neglect it as well as all the last terms of the formulae (7).

The problem which we want to solve is to express Δ in Δ_R, Δ_R representing the value of Δ on the sphere. It is well known that for a point outside the attracting masses, T can be written in the shape:

$$T = \frac{k^2}{\varrho}\left[\frac{K_2}{\varrho^2} + \ldots . + \frac{K_n}{\varrho^n} + \ldots .\right] . \quad . \quad . \quad . \quad . \quad (10^A)$$

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[1]) In the original paper (Kon. Akad. v. Wet. Amsterdam, Dl. 37 N^0. 1) the second term of the right member of (7^D) has been neglected. The reasoning remains the same when taking it into account.

in which k^2 is the gravitational constant and $K_2 \ldots K_n \ldots$ are spherical harmonics of the $2^{nd} \ldots n^{th}$-degree. There are no spherical harmonics of zero and first degree, because the total mass of the actual and of the theoretical earths are the same, so that the differential masses, which cause the potential T, have a total mass zero, and because the centres of gravity of the masses of the actual and the theoretical earths coincide with the origin of the radius ϱ.

We have now:

$$g - \gamma = -\frac{\partial T}{\partial \varrho} = \frac{k^2}{\varrho}\left[3\frac{K_2}{\varrho^2} + \ldots + (n+1)\frac{K_n}{\varrho^n} + \ldots\right] . \quad (10^B)$$

$$\Delta = -\frac{\partial T}{\partial \varrho} - 2\frac{T}{\varrho} = \frac{k^2}{\varrho^2}\left[\frac{K_2}{\varrho^2} + \ldots + (n-1)\frac{K_n}{\varrho^n} + \ldots\right] . \quad (10^C)$$

and for $\varrho = R$:

$$\Delta_R = \frac{k^2}{R^2}\left[\frac{K_2}{R^2} + \ldots + (n-1)\frac{K_n}{R^n} + \ldots\right] . \quad (10^D)$$

$$T_R = \frac{k^2}{R}\left[\frac{K_2}{R^2} + \ldots + \frac{K_n}{R^n} + \ldots\right] . \quad (10^E)$$

When Δ_R is known we can develop Δ_R into a series of spherical harmonics, which gives us $K_2 \ldots K_n \ldots$ and by substituting these values in the formula for Δ, the problem is solved because formula (10^C) is convergent when $\varrho \geqq R$. We can however deduce an equation which allows a simpler computation, by expressing Δ directly in Δ_R. This gives besides a more useful formula for the case Δ_R is not completely known over the whole globe, which is of course the actual state of things. The deduction is quite analogous to the method of STOKES, who solved the problem of computing T_R (and thereby $N_R = T_R/\gamma$) from Δ_R, by expressing T_R directly in Δ_R, so that the elaborate way of first computing all $K_2 \ldots K_n \ldots$ can be avoided.

The spherical harmonic of the n^{th} degree of Δ_R is:

$$\frac{2n+1}{4\pi}\int_0^\sigma P_n \Delta_R \, d\sigma . \quad (11)$$

in which: $d\sigma$ = surface-element corresponding to Δ_R of a sphere with radius 1.

P_n = LEGENDRE's spherical harmonic of the angle ψ between the radius ϱ of the point A where we want Δ and the radius R of the point P where we have Δ_R.

This gives:

$$\Delta = \sum_2^\infty \left(\frac{R}{\varrho}\right)^{n+2} \times \frac{(2n+1)}{4\pi}\int_0^\sigma P_n \Delta_R \, d\sigma$$

which is convergent because $\varrho \geqq R$. Therefore:

$$\triangle = \frac{1}{4\pi} \int_0^\sigma \left[\sum_2^\infty (2n+1) P_n \left(\frac{R}{\varrho}\right)^{n+2} \right] \triangle_R \, d\sigma$$

or

$$\triangle = \frac{1}{4\pi} \int_0^\sigma S_\triangle \triangle_R \, d\sigma \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12^A)$$

with

$$S_\triangle = \sum_2^\infty (2n+1 \, P_n \left(\frac{R}{\varrho}\right)^{n+2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12^B)$$

Now $S_\triangle$ can be expressed in R, ϱ and ψ by making use of the well-known formula:

$$\sum_2^\infty P_n \left(\frac{R}{\varrho}\right)^n = \frac{\varrho}{r} - 1 - \frac{R}{\varrho} \cos\psi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

in which r is the distance AP, given by:

$$r = (\varrho^2 - 2\varrho R \cos\psi + R^2)^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

Multiplying (13) by $\left(\frac{R}{\varrho}\right)^{\frac{1}{2}}$, differentiating according to $\frac{R}{\varrho}$ and multiplying by $2\left(\frac{R}{\varrho}\right)^{\frac{1}{2}}$ makes the first member equal to $S_\triangle$, and we find:

$$S_\triangle = \frac{R^2(\varrho^2 - R^2)}{\varrho r^3} - \frac{R^2}{\varrho^2} - 3\frac{R^3}{\varrho^3} \cos\psi \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

(12^A) combined with (15) gives $\triangle$ expressed in $\triangle_R$.

Still we have to be careful if we want to use these formulae for determining the difference between $\triangle$ and $\triangle_R$: If $\triangle_R$ does not contain spherical harmonics of zero and first degree as has been assumed, the difference can indeed be found simply by subtracting $\triangle_R$ from formula (12^A), but if in the formula for $\triangle_R$ those spherical harmonics are not zero, we must first subtract those terms from $\triangle_R$ before taking the difference with $\triangle$. $\triangle$ is certainly free from those terms even if $\triangle_R$ is substituted in (12^A) without correction, because $S_\triangle$ does not contain spherical harmonics of zero and first degree. The terms of zero and first degree in $\triangle_R$ are easily found by applying formula (11) and in this way we find for the difference $\triangle - \triangle_R = \delta$:

$$\delta = \frac{1}{4\pi} \int_0^\sigma S_\delta \, \triangle_R \, d\sigma - \triangle_R \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16^A)$$

with:

$$S_\delta = \frac{R^2(\varrho^2 - R^2)}{\varrho r^3} + \frac{(\varrho^2 - R^2)}{\varrho^2} + 3\frac{(\varrho^3 - R^3)}{\varrho^3} \cos\psi \quad . \quad . \quad . \quad . \quad (16^B)$$

in which formulae $\triangle_R$ may be substituted as it is, without taking away its terms of zero and first degree. δ will not contain those terms.

Formula (16^B) converges towards zero when $\varrho - R$ is converging towards zero. If the difference $\varrho - R$ which we will represent by h, is small, we can write, neglecting second and higher powers of h in S_δ:

$$S_\delta = \frac{h}{R}\left[2\frac{R^3}{r^3} + 2 + 9\cos\psi\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16^C)$$

Applying these formulae to the geoid in stead of to a sphere with radius R, we have to substitute for R the mean earth's radius and for h the elevation above the geoid.

For giving an idea of the magnitude of the variation of $\triangle$ outside the geoid, we will apply the formulae to a special case.

Supposing a circular patch of anomalies, defined by the formula:

$$\triangle_R = \triangle_C\left(1 - \frac{u^2}{l^2}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17^A)$$

in which $\triangle_C$ is the value in the centre, u the horizontal radius from $\triangle_R$ to this centre and l the outer limiting radius. Suppose l small with regard to the earth's radius. In this case we can neglect the second and third term of (16^C). We find for a point at an elevation h above the centre:

$$\delta_h = -2\frac{h}{l}\left[\sqrt{1 + \frac{h^2}{l^2}} - \frac{h}{l}\right]\triangle_C \quad . \quad . \quad . \quad . \quad . \quad (17^B)$$

For $h = 0.36\ l$ we get $\delta_h = 0.5\ \triangle_C$, that is to say that the anomaly has diminished to half its value for an elevation about one sixth of the diameter of the patch.

This allows the conclusion, that, generally speaking, the effect of the diminution of $\triangle$ with the elevation is too small to warrant a corresponding reduction in bringing back the result of a gravity determination to sea level: it will seldomly exceed 0.001 or 0.002 cm sec^{-2} per 1000 m elevation.

The formulae (12), (15), (16) and (17) may be applied to all other quantities which can be represented by a series like:

$$q = \frac{C}{\varrho^2}\left[a_2\frac{K_2}{\varrho^2} + \ldots + a_n\frac{K_n}{\varrho^n} + \ldots\right] \quad . \quad . \quad . \quad . \quad (18)$$

in which c, $a_2 \ldots a_n \ldots$ are constants and therefore independent of the radius ϱ or the angle ψ. We have simply to substitute q and q_R for $\triangle$ and $\triangle_R$. In this way they may for instance be used to express the variation of $g - \gamma$ outside the geoid in $g_0 - \gamma_0$ on the geoid, $g - \gamma$ representing the difference of the actual and the theoretical gravity in the same point.

§ 3. *Potential outside the geoid.*

To express the variation of T outside the geoid in T_0 on the geoid, we have analogous formulae deduced in the same way:

$$T = \frac{1}{4\pi} \int_0^\tau S_T \, T_0 \, d\sigma \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19^A)$$

$$S_T = \frac{R\,(\varrho^2 - R^2)}{r^3} - \frac{R}{\varrho} - 3\frac{R^2}{\varrho^2} \cos\psi \quad . \quad . \quad . \quad . \quad . \quad (19^B)$$

For the difference $t = T - T_0$ we find:

$$t = \frac{1}{4\pi} \int_0^\sigma S_t \, T_0 \, d\sigma - T_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (20^A)$$

$$S_t = \frac{R\,(\varrho^2 - R^2)}{r_3} + \frac{(\varrho - R)}{\varrho} + 3\frac{(\varrho^2 - R^2)}{\varrho^2} \cos\psi \quad . \quad . \quad . \quad (20^B)$$

and for a small elevation h:

$$S_t = \frac{h}{R}\left[2\left(\frac{R}{r}\right)^3 + 1 + 6\cos\psi\right] \quad . \quad . \quad . \quad . \quad . \quad (20^C)$$

For expressing the distance N in some outside point between corresponding potential surfaces of the actual and theoretical earths, we have to substitute for T: γN and for T_0: $\gamma_0 N_0$.

For local irregularities in T or N of an extension, which is small with regard to the earth's radius, we can neglect the second and third terms of (20^C) and we find the same formula for the diminution with h as for $\triangle$. This is also true for the general case, if $\triangle_R$ and T_0 are free of spherical harmonics of zero and first degree, because the difference between S_3 and S_t contains only spherical harmonics of zero and first degree.

Lastly we will ask to express T outside the geoid in the anomaly $\triangle_0$ on the geoid, or if we replace again the geoid by the sphere with radius R, to express T in $\triangle_R$. This is an enlargement of the problem of STOKES; by making the radius ϱ of the point, where we want T, equal to R, it is brought back to the identical problem.

Following the same way of deduction as for the formula for $\triangle$ we get:

$$T = \frac{R}{4\pi} \int_0^\sigma S_{\triangle T} \, \triangle_R \, d\sigma \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21^A)$$

with

$$S_{\triangle T} = \sum_2^\infty \frac{(2n+1)}{(n-1)} P_n \left(\frac{R}{\varrho}\right)^{n+1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (21^B)$$

and we find $S_{\triangle T}$ by multiplying (13) by $\left(\frac{R}{\varrho}\right)^{\frac{1}{2}}$, differentiating according to $\frac{R}{\varrho}$, multiplying by $2\left(\frac{R}{\varrho}\right)^{-1\frac{1}{2}}$, integrating according to $\frac{R}{\varrho}$ and multiplying by $\left(\frac{R}{\varrho}\right)^{2}$:

$$S_{\triangle T} = 2\frac{R}{r} + \frac{R}{\varrho} - 5\frac{R^2}{\varrho^2}\cos\psi - 3\frac{Rr}{\varrho^2} - 3\frac{R^2}{\varrho^2}\cos\psi \log nat \frac{[\varrho - R\cos\psi + r]}{2\varrho} \quad (21^c)$$

By putting $\varrho = R$, which gives $r = 2R \sin \frac{1}{2}\psi$, we get back to the formula of STOKES. The distance N is of course found by dividing T by γ. In this way the formula gives the outside potential surfaces of the earth, when the gravity anomaly on the geoid is known and provided the theoretical outside potential surfaces have been computed. N may of course be differentiated in the same way as has been done in the first paragraph; we find then the deflection of the plumb-line in a point outside the geoid, expressed in the gravity anomalies on the geoid, or in their horizontal gradient. Executing the same thing with the formula for N deduced from formula (19), we can get this plumb-line deflection expressed in the value of N_0 on the geoid or in the plumb-line deflections on the geoid.

§ 4. *The validity of the formula of* STOKES.

To find the order of magnitude of the neglected terms of the formula of STOKES, we suppose a fictitious earth with a mass equal to the total mass of the real earth, and of which the outside potential surface is a sphere with a radius R. The difference of R and the earth's radius is of the order of the flattening. The gravity on this sphere is, according to a potential theorem, constant over the whole surface.

To this fictitious earth is added a mass-distribution of positive and negative masses with a zero total mass, in such a way, that the combination of these masses with the fictitious earth gives a geoid on which the gravity anomalies (i.e. the gravity minus the above mentioned constant value) are the same as the gravity anomalies $\triangle_0$ of the real earth in corresponding points of the real geoid; for corresponding points we may for instance take points with the same geographical coordinates. We will henceforth indicate this added mass-distribution with the letter M.

If $\triangle_0$ is supposed to be known, the formulae of §§ 2 and 3 allow the complete determination of the outside gravity-field of this mass-distribution M without any further neglections than those, given by the last terms of the formulae (7); in the same way the formula of STOKES may give the distance N_R between the sphere and the geoid of the above combination without neglecting more than these terms of (7).

We will now combine the mass-distribution M with the theoretical earth, which has been defined in the beginning of § 2, and of which

the outside potential surface is the spheroid. This addition to the theoretical earth causes the potential surface to shift from the spheroid to a geoid, which will nearly coincide with the real geoid. If we suppose Δ_0 to be known, the distances N'_0 between this geoid and the spheroid, and the anomaly Δ'_0 on this geoid, may be computed with the formulae of §§ 2 and 3; we have to introduce in these formulae for h the distance between the speroid and the sphere.

We will now prove that the difference $\delta' = \Delta'_0 - \Delta_0$ is of the order of gf^3, i. e. of the order of the flattening with regard to Δ_0 itself. If this is true we may neglect δ' as we have already neglected quantities of the same order in formula (9); these neglections are insignificant considering the accuracy of the determination of the gravity anomalies. And secondly; if we neglect the difference between the anomaly Δ'_0 and the real anomaly Δ_0 we may confound the geoid, which we have got by combining M with the theoretical earth, with the real geoid, so that the distance N'_0 can be considered to be also valid for this last geoid.

We need not doubt that δ' is of the order of gf^3 as far as δ' is given by the second and third terms of (16^C) in combination with (16^A), because $\frac{h}{R}$ is of the order of the flattening, while we assumed that Δ_0 is of the order of gf^2. The only doubt, which might arise, concerns the effect of the first term of (16^C) for small r. We have seen in § 2, page 13, that because of this term, the difference δ, caused by local anomalies, may get a value of the order of $\frac{2h}{l}\Delta_0$, in which l is the horizontal extension of the anomaly. So we see that, locally, values of δ' may occur, exceeding the order of magnitude of gf^3. It is clear that these local values of δ' may be neglected for the determination of N'_0 in some point A of the spheroid which is far away; a more thorough investigation which we will not repeat here, confirms this opinion. The question is however, if they have no effect if they occur near to A.

In order to prove that this is not the case, we will suppose that our sphere with radius R is tangent to the spheroid in A, while the radius is supposed to coincide with the smallest radius of curvature of the spheroid in that point (see the supposition about this radius on page 1), so that the whole sphere is inside the spheroid. This is necessary if we want to apply the formulae of the previous paragraphs, because we assumed there: $\varrho \geqq R$. This supposition makes it of course impossible that the centre of the sphere should coincide with the centre of gravity of the earth, but it may easily be seen, that this only affects the deduction in this way, that ϱ in formula (8) for $\frac{\partial\gamma}{\partial\varrho}$ is not measured from the centre of gravity of the earth, but from the centre of the sphere at some distance of the order of fR. This means a deviation of the formula (9)

of the order of gf^3, which does not exceed the term, which has already been neglected.

If we introduce in A an azimuthal coordinate α, representing the angle between the vertical plane through A in which the sphere and the spheroid osculate, and the vertical plane through A and through the point P, where we suppose that the value of δ' occurs, we find in P a distance h between the sphere and the spheroid, which may approximately be given by:

$$h = (f)\,\frac{r^2}{R}\sin^2\alpha$$

in which (f) means a constant of the order of the flattening, while r represents again the distance AP.

We find thus that δ' is of the order of:

$$(f)\,\frac{r^2}{Rl}\times\triangle_P$$

in which $\triangle_P$ is the value of $\triangle_0$ in P. We see therefore that, even for small r, we need not fear that δ' would exceed the order of gf^3.

We may conclude that, if we neglect in $\triangle_0$ quantities of the order of gf^3, our problem is brought back to the determination in A of the distance N'_0, corresponding to the mass-distribution M, which has been defined in the beginning of this paragraph. As A is also a point of the sphere, N'_0 equals N_R, so that it can be computed by applying the formula of STOKES: we have to substitute in this formula the anomalies $\triangle_0$ and the radius R. We may notice however, that it will doubtless be somewhat better to substitute in this formula for R the mean earth's radius; we only chose a slightly different value for R in order to be able to apply the formulae of the previous paragraphs.

The conclusion at which we arrive, is that we are justified in using the formula of STOKES: the neglections in N will not exceed the order of Rf^3, i. e. one metre. We may apply the formula of STOKES as well for the determination of the general shape of the geoid as for the deduction of its local shape and obviously we may follow the same reasoning and arrive at the same conclusion for the formulae, derived from the formula of STOKES by differentiating it once or twice according to a direction tangent to the geoid.

Physiology. — *Annotations on the physiology and the anatomy of a dog, living 38 days, without both hemispheres of the cerebrum and without cerebellum.* By Dr. G. G. J. RADEMAKER and Dr. C. WINKLER.

(Communicated at the meeting of April 28, 1928).

Until now, different workers have not succeeded in keeping long enough alive a dog, after the removal of both hemispheres and cerebellum, to do physiological and anatomical researches on it.

Such researches may have a great interest for :

1°. a physiology of the higher parts of the central system, wanting to be freed from the psychological nomenclature in which it is, necessarily at present, bound ;

2°. an anatomy, wanting to know the systematic architecture of those higher parts, which till now, is only known very roughly.

Dr. RADEMAKER has tried to produce such animals for research, forced by considerations, from which he gives the following report.

Total or partial removal of the cerebellum in normal young animals has not answered in a sufficient way the question : what function may be practised by the cerebellum ?

It was his endeavour to solve this question, by removing the cerebellum in animals without hemispheres.

It could be expected, that the loss of all conditioned reflexes after extirpation of the hemispheres might simplify the symptoms of the removal of the cerebellum.

Firstly, the comparison of the behaviour of animals without hemispheres before and after the loss of the cerebellum should probably produce more distinct facts to enable us to understand the cerebellar function.

Secondly : the study of the compensation of the symptoms produced by cerebellar removal in normal animals, compared with those after such removal in animals without hemispheres, should open the possibility of determining the part, played by the hemispheres in compensating disorders caused by cerebellar removal.

For most workers believe, that the transient cerebellar disorders, which gradually pass to a certain extent, after cerebellar removal in normal animals, are compensated for by the function of the hemispheres, though nobody has made special experiments upon this question.

And lastly. Only by removal of the cerebellum after that of the hemispheres, can it be for certainty fixed, which function the brainstem may perform without being influenced by cerebrum and cerebellum.

The dog, now presented to you, is an animal, living 38 days without

hemispheres and without cerebellum. At the first operation, on December 9th 1926, 97 days before death, the left hemisphere has been taken away; 20 days afterwards, on December 29th 1926, 77 days before death, the right hemisphere was extirpated, and the 7th of February 1927, 38 days before death, the cerebellum was removed.

All operations were very well supported. Difficulties in the regulation of the warmth of the animal, of its feeding, of its uncleanliness were overcome by the well-trained nursery-knowledge and the devotion of Mrs. RADEMAKER. But a fortnight before death, an infectious distemper of dogs made its entrance into the kennels. Fever, purulent rhinitis and conjunctivitis, frequent respiration, coughing and sneezing occurred. The animal died at the 17th of March 1927. At the autopsia were found, purulent bronchitis and pneumonia and an empyema of the right frontal sinus.

In the short time in which research was possible, different interesting facts were noticed.

After the removal of the left hemisphere, the animal walked mostly in a circle to the left, counter-clockwise.

The preference for walking to the left, persisted after the removal of the right hemisphere, contrarely to what is ordinarily seen. This had been the only asymetrical disorder seen in the animal after the removal of both hemispheres.

It is difficult to prove, that this symptom was caused by the removal of the striatum at the left side and the nearly absolute sparing of the striatum at the right side, as the anatomy taught us (fig. 43—96 of the series).

After the extirpation of both hemispheres the animal had the ordinary symptoms, described by many investigators.

1°. the animal is blind, deaf and cannot smell, if one regards the extensive reactions, which the normal animal produces on optic, acustic and olfactive stimulations, as a measure for seeing, hearing and smelling.

2°. the animal walks practically as a normal dog walks, setting his paws on the earth in the proper position and making no errors in keeping his equilibrium (fig. 1).

3°. the animal has lost all the reactions of position of the legs, and all the reactions of correction made by the extremities, originated by the surface of its body (fig. 2).

4°. the supporting-tonus of the hind-legs has much diminished. They bend promptly at a slight pression upon the pelvis or when a sandbag of 2 kg is put on the back of the animal. But the tonus of support has not altered in the forelegs.

Now, as in this animal without hemispheres, the removal of the cerebellum was executed, the same disorders were seen as in a normal animal after the decerebrate section of SHERRINGTON. The animal had stiffened. Without trying to stand up, the animal placed on its back, retained this position with its neck stretched backward, with its stiffened back bent convexely, and with the four extremities totally stretched and

stiffened, resisting violently every passive flexion. The stretched tonus was the so-called plastic tonus; it presented the clasp-knife phenomenon and did not disappear after passive flexion of the distal joints of the extremity. After a week, this rigidity could still be seen, in the second-third week it disappeared (fig. 3).

After varying the position of the head against the trunk and still more after varying the position of the head in space, the extension-rigidity could be seen diminishing or augmenting (tonic cervical- and labyrinthic-reflexes of MAGNUS and DE KLEYN).

However, at the end of the first week, the extension-tonus of the hind-legs still being augmented, only a very slight supporting-tonus could be shown in them. A pressure of 2 kg exerced on the sole was sufficient to bend them (fig. 4).

On the contrary a pression of 10 kg, exerced on the sole of the foreleg was supported, without bending, and if a sandbag of 5 kg was placed upon the shoulders, the forelegs did not bend (the dog's weight was 6.5 kg) (fig. 4).

In the course of the second to the fourth week after the cerebellar removal, the rigidity disappeared and the animal made attempts to right himself.

The labyrinth-righting-reflexes returned, the supporting-tonus of the hind-legs augmented, positive and negative reactions of support and different other reactions of the extremities were seen again.

If the animal was put on his hind-legs and the trunk moved in a backward direction, the animal made an alternative movement backward with the hind-legs, and the moving of the trunk to the left or to the right, was followed by a running movement to left or to the right, of the hind-legs.

Also, if the animal only was supported by one hind-leg, this leg followed the movement of the trunk forward, backward, to the right or to the left, by hopping in the same direction, as the movement of the trunk was made.

If the animal was held in the air by the shoulders, and then was permitted to descend on an oblique plane (with the back-part of its body), the hind-legs adjusted to the inclination of the plane and the animal walked exactly up or down the incline as was desired (fig. 5).

The described hop-movements of the legs play, in normal animals, an extensive part in keeping and in restoring equilibrium. The dog *Robbie* however, if placed, at that time free on his legs, always fell on the ground, because the hopping movements of the hind-legs came much too late and those of the fore-legs were not yet restored.

If, at this stage, the animal was placed on the left side, in its cage, it soon lifted the head and the forepart of the body and remained in a rolled-up position in the manner of a normal dog.

Sometimes the animal tried to take an upright position, but, as the hind-part of the body remained in a reclining position, it turned round in clock-wise-movements to the left. Sometimes, the animal succeeded in getting, by

a leap, on its four legs, but always then it fell to the left side. It never lifted the head and fore-part of the body so well from the left side, as it did from the right side.

The animal did not govern well the movements of the head. If it was free in the air, the head fell backward or lateral-ward. If the head was damaged by this movement at the bars of its cage, the animal could whine vehemently.

The restitution of different movements of the extremities in dog *Robbie* as f.i. the hopping movements after trunk-motion, the running backward or foreward of the hind-legs according to trunk-motion — all movements also missing in the first days after cerebellum-extirpation in normal dogs — proves, that they may be restituted without intervention of the hemispheres.

Still other symptoms of the animal are worth mentioning. If chopped-meat was placed in the mouth of the animal, it was well masticated and swallowed. But crusts of bread were rejected from the mouth. And if meat and crusts of bread were given together, the meat was swallowed, the bread-crusts rejected.

If the animal was irrigated with water, it shook itself like a normal dog. If its nose was moistened, the dog licked it off. Tickling of the mucous membrane of the nose was answered by sneezing.

After turning the animal along a dorso-ventral axis, vivid eye-nystagmus is seen.

Interesting was the intense reaction made by the animal on accustic stimuli. The answer to the sharp sound, made by sucking in the air with closed lips, was: adjusting of the auricles, lifting of the head and a movement with the legs. The animal without hemispheres answered this stimulus more intensely, after the removal of the cerebellum than before.

Many tactile stimuli gave vivid reflexes; so the cornea-reflexes were found, as also the movements of the eyelids after touching of the lashes, and movements of the auricles after touching of its hairs.

The animal discharged spontaneously urine and faeces. In the urine neither albumines, nor glucose was found, even in the first days after the cerebellar-removal.

The animal lived in two alternatingly varying periods. In one it was restless, it tried to make walking movements and to lift itself. In the other period it seemed to sleep. If shaken or pinched, it opened its eyes, yawned and moved its hairs as a dog that is awaking.

And lastly it must be mentioned, that the animal, after being set upon its legs, never had "uncontrolled" (LUCIANI's astasia) movements. Those movements do not appear immediately but only after some time, if the cerebellum-removal in normal animals with intact hemispheres has taken place.

Reckoning with the short space of time lived by the animal after the last operation, and with the appearance of an infectious distemper in the third week after it, the conclusion is nearly certain, that the animal has not at all attained the maximum of compensation, and that the possible

restoration of the post-operative phenomena might have been much more evident.

The anatomical annotations upon the dog *Robbie*, are the following.

1°. If comparing the rest of *Robbie's* brain with normal brains of a dog, with the naked eye on a photo (fig. 6), it is not only seen that the removed nervous mass is very important, but also that the border lines of the removed hemispheres demarcate the brainstem at the left and at the right side in a very unequal way.

2°. It seems that the left hemisphere has been removed more completely than the right one. The remaining part of the right hemisphere surpasses widely that of the left in a frontal direction. The most frontal piece of the right hemisphere is transformed in a spherical mass of nervous tissue, connected with the dorsal wall of the brainstem, which at the right side is nearly twice as broad as on the left side (fig. 6). The cerebellum is taken away on both sides, except a few lateral lamellae only slightly connected by fibres with the brainstem.

3°. Research by uninterrupted series of sections gives a solution of the difference between the left and right hemisphere.

At the left side, the striatum has been completely removed with the extirpation of the pallium. On the contrary on the right side, the striatum has been spared. Only at its most frontal end it possibly may be a little damaged.

A more detailed research makes it obvious, that the sphaerical mass of nervous system (fig. 7, N°. 27 of the series) contains the dorsally opened frontal end of the lateral ventricle, with its medial and lateral walls. The latter is connected with the striatum (fig. 7, N^{os} 27 and 69 of the series) in which all the constituant parts, nucleus caudatus, nucleus lentiformis with its putamen and globus pallidus and the nucleus accumbens septi are found. The operating knife has gone lateral to the external capsule towards the olfactory convolutions (fig. 7, N^{os} 69, 83, 96 of the series).

The striatum is very rich in fibres (fig. 7, N^{os} 83—111). This may be compared with the status marmoratus, which OSCAR and CÉCILE VOGT have described in pathological cases of the striatum in overfunction.

It is sending out a very strong system of fibres along the "Kammsystem of EDINGER" and the ansa lenticularis (fig. 7, N^{os} 111 and 126), crossing the capsula interna, wherein the cortical fibres are strongly degenerated but not all lost.

At the left side neither a "Kammsystem" nor an ansa lenticularis is found. Together with the totally removed striatum, they have disappeared at that side.

At the right side a part of the pyriform gyrus with the nucleus amygdalae has remained and from there a certain number of fibres enter well-myelinated in the anterior commissure, that has been totally lost at the left side.

At the left side, the small remainder of the striatum which has been spared (fig. 7, N⁰. 83 of the series), appears as a small field of circular form between brainstem and the left corpus callosum. In this field some fibres appear, being fibres of the capsula interna.

In this way the experimental border-line between removed cortex + striatum and the remaining diencephalon is found (fig. 7, N⁰. 96 of the series) as composed of three parts. In the middle layer is seen the internal capsule, ventrally the cornu AMMONIS with fimbria and columna fornicis, at the dorsal end the corpus callosum, with the sulcus medialis of the hemispheres (fig. 7, N^os 96 and 111 of the series). They are united in a mass of fibres, forming the caudal border of the cicatricial line (fig. 7, N⁰. 111 of the series), where it ends.

The first section, in which no longer operation-lines are passing through the hemispheres is in fig. 7, N⁰. 126 of the series. There it is seen, that all fibres going from the striatum into the "Kammsystem" or ansa lenticularis have fallen out; neither are they found in more caudal sections. Whereas at the right side those fibre-systems have developed very intensely and are hypertrophied. Dr. MORRISON of Boston is working out the sequences of the loss of those systems at the left side.

However, it may already be mentioned here, that, influenced by the well-preserved striatum of the right side, there are found cell-territories in the middle of the right diencephalon, with well preserved middle-sized and small nerve-cells, missing completely in the left, where the striatum has been removed (fig. 8).

Both diencephala are severely damaged by the removal of both hemispheres and have lost the greater part of their cells. But in the middle of the left diencephalon, in the nucleus medialis and in a medial part of the nucleus ventralis, are seen only holes, where formerly cells have been, while, in the same region of the right diencephalon are found well preserved cells (fig. 8).

To this region the frontal radiation of the red nucleus can be followed (to the diencephalon) and there are reasons for accepting, that at the right side, going from the red nucleus to the thalamus, from there *to the nucleus caudatus* and farther to the nucleus pallidus, the Kammsystem and ansa lenticularis, a system has remained intact and functioning.

At the left side cells in the substantia nigra have disappeared (fig. 9). At the right side (fig. 9), those depending from the striatum, and they are the majority, have all remained, only those depending from the cortex, have disappeared. In the corpus subthalamicum of the left side many of the larger cells in the lateral part have disappeared, while nearly none in the medial parvocellular part of this nucleus are missing.

It would have been of great interest, if the degenerated and fallen out systems between the left striatum and the tegmentum of the pons VAROLI, medulla oblongata and spinalis, as well as the strongly developed paths from the right striatum and the corresponding parts of the nervous system,

had not been complicated by the presence of the cortical and cerebellar pathways. But the animal has not lived long enough to bring a total loss of the cortical and cerebellar fibres.

This may be illustrated by the study of the restiform body and the brachium pontis of the dog *Robbie,* compared to those parts in animals living 136 days or 360 days without a cerebellum.

In all those animals the cell-preparations of the nervous system do not differ very much. In the spinal cord, large cells in the posterior horns and most in the columns of CLARKE are all degenerated after 38 days, and after 136 days or 360 days have all disappeared. In the medulla oblongata in all cases the cells are degenerated and most of them have disappeared in the nuclei olivares inferiores, the nuclei funiculi laterales, in the nuclei proprii corporis restiformis, together with some of the smaller cells in the nuclei of GOLL and BURDACH. In the ventral formation of nuclei in the pons VAROLI all cells have disappeared after 38, 136 and 360 days.

But fibre-preparations differ intensily of cell-preparations after 38 days or longer, because the degeneration of fibres follows the degeneration of cells, but the loss of fibres follows the loss of cells in a much slower tempo.

In dog *Robbie,* WEIGERT-preparations give the impression that the corpus restiform and the brachium pontis are unaltered.

After 136 and 360 days all fibres in the restiform body have disappeared except the long dorsal spino-cerebellar pathway, which never disappears totally.

And the brachium pontis in *Robbie,* seems to have no fibres missing, it contains all fibrae transversae and all fibrae rectae pontis, whereas after 136 or 360 days, all fibres in the brachium pontis, together with the fibrae transversae et rectae, have disappeared with the cells.

We have not been able to have in *Robbie,* the fibre-systems of the right striatum, well developed but un-complicated, because not all cortical and cerebellar fibres had been destroyed. This would have been possible, if the animal had lived longer.

Fig. 1. The dog *Robbie*, after removal of both hemispheres of the brain. Exposition on the 5th of February 1927.

Fig. 2. Absence of reflexes of posture after removal of both hemispheres of the brain. If the head or the ventral surface of the body were touching a table, the extremities were not placed upon the table.

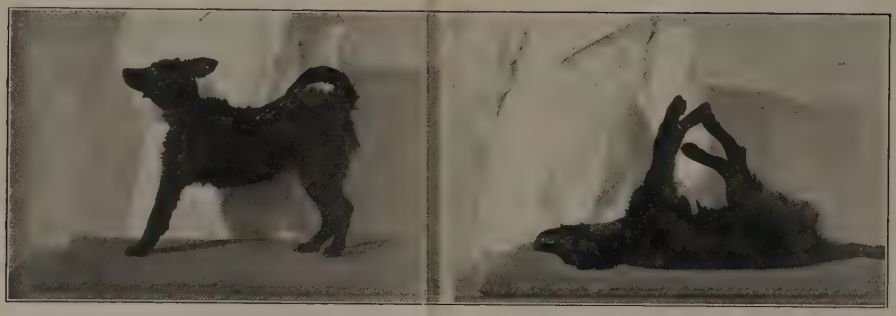


Fig. 3. The dog *Robbie*, one week after the removal of the cerebellum. The animal has extension-tonus of the extremities on both sides.
Exposition on the 13th of February 1927.

Fig. 4. 1. The hind-legs are bending at a pressure upon the soles of $1^1/_2$ kilogram.
2. The fore-legs resist bending at a pressure upon the soles of 11 kilogram.
3. If a sandbag of $5^1/_2$ kilogram is hung on the shoulders of the animal, the fore-legs do not bend. Weight of the dog $6^1/_2$ kilogram.
Exposition on the 13th of February 1927.

Fig. 5.

1. The animal is lowered with the hind-part of its body upon an inclined plane.
2. The animal does not sit, but, by moving its legs backward, it tries to remain standing and walks down the inclined plane.

3 and 4. Here also the animal is lowered with the hind-legs upon an inclined plane. It again tries by moving its legs backward to keep from sitting and so walks backward up the inclined plane.

Exposition on the 23th February 1927.

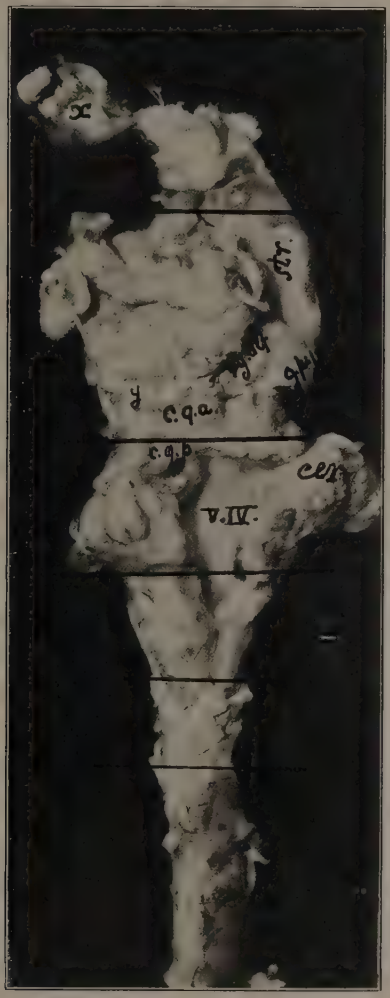


Fig. 6.
The rest of the brain of the dog *Robbie* seen from a dorsal view.

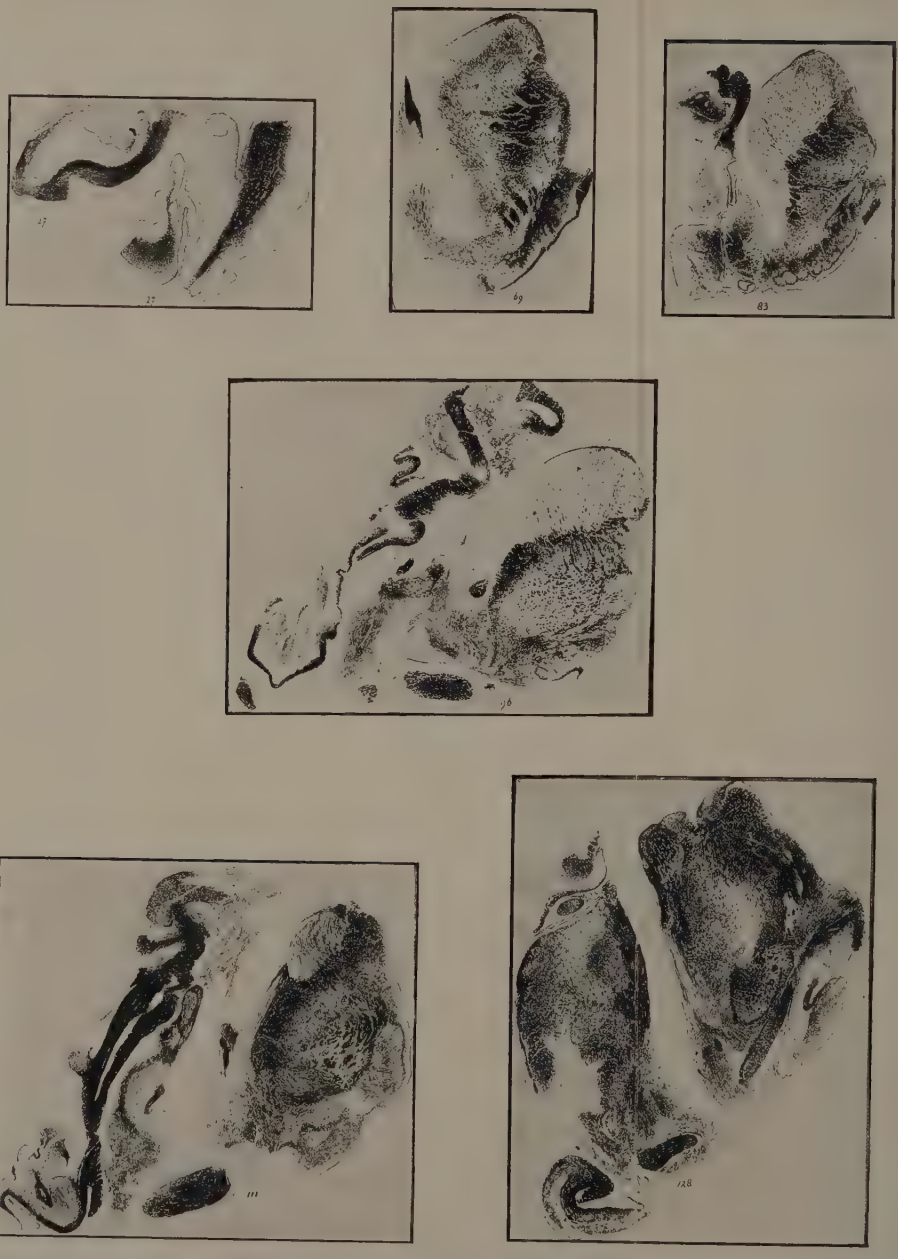


Fig. 7.

6 drawings of sections through the frontal end of the brain of dog *Robbie*.

At the left side the striatum is removed.

At the right side the striatum has been nearly totally spared.

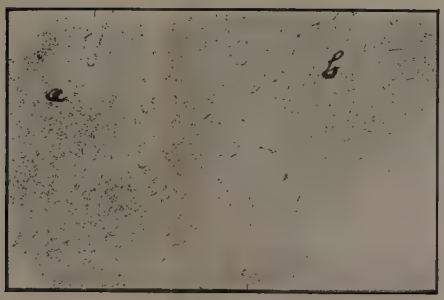


Fig. 8.

Transverse section through the commissura media thalami.

At the left: in *a*. total degeneration of cells in the medial part of the thalamus.
At the right: in *b*. several well preserved cells are found in that medial part.

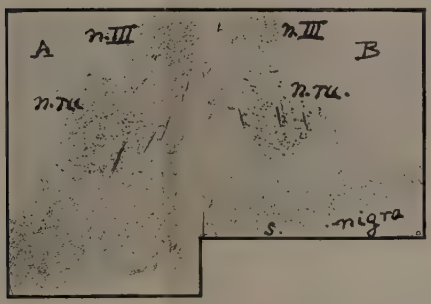


Fig. 9.

Two sections through the pedunculus cerebri.

A. At the left: loss of nearly all cells in the substantia nigra.
B. At the right: nearly all cells in the substantia nigra are well preserved.

Anatomy. — *On the Developmental History of the müllerian duct in the Sterlet (Acipenser ruthenus).* By W. MOOY. (Communicated by Prof. J. W. VAN WIJHE.)

(Communicated at the meeting of February 25, 1928).

As an assistant and under the supervision of Prof. J. W. VAN WIJHE I have studied the early development of the Müller's duct in Acipenser ruthenus.

I was prompted to make this inquiry after a publication of A. OSTROUMOFF. However this worker wanted the first stages in the development of Müller's duct, Prof. VAN WIJHE succeeded in providing me with the missing stages.

OSTROUMOFF (1908) examined fishes to the length of 18 mm in which he could not detect anything of the oviduct. The following stages up to that of 35 mm were missing in his material. In that of 35 mm he recognized the müllerian duct, in which he distinguished 3 parts:

1°. the front-end, in the form of a thickened stripe of peritoneal epithelium, in his opinion a rudiment indicative of a phylogenetic relation to the posterior margin of the pronephros.

2°. a median part, the ostium abdominale passing into

3°. a channel developing posteriorly as a solid foundation between the peritoneal epithelium and the wolffian duct.

Furthermore I found in the literature on this subject the following particulars:

FELIX (1906) writes: "Unbekannt ist die Eileiterentwicklung der Ganoiden und Dipnoer". OSCAR HERTWIG (1910) does not express himself about the development of Müller's duct in Ganoids.

The most interesting and at the same time the most recent study on this subject is that of MASCHKOWZEFF (1924). He writes:

"Der grosze Eileitertrichter bildet sich aus einigen zusammengeschmolzenen sekundären Nephrostomaltrichtern des Mesonephros. In kaudaler Richting von der Wurzel des Eileitertrichters an wachsen die aus Auswüchsen der sekundären Baumanschen [1]) Kapseln entstandenen sekund. Nephrostomalkanälchen (N_2) ebenso unter dem Harnleiter; sie bilden aber keine offene Nephrostomaltrichter, sondern laufen unter das Peritoneum der Rumpfhöhle blind aus.

Durch Verschmelzung dieser blinden sekundären Nephrostomaltrichter

[1]) MASCHKOWZEFF writes mistakenly „Bauman", instead of Bowman.

(N_2) bildet sich ein Zellenband, das vorn in den Eileitertrichter übergeht.

In der Nähe des Eileitertrichters bildet sich eine Höhlung innerhalb des kompakten Zellenbandes, das sich somit zu einem Kanal umgestaltet (Abb. 6, El.).

Das Wachstum des Eileiters in kaudaler Richtung geht ziemlich langsam vor sich und im steten Verhältnis zu der Abbildung der sekundären Baumannschen Kapseln (M_2) und der sich aus diesen entwickelnden sekundären Nephrostomalkanälchen und seiner blindgeschlossenen Nephrostomaltrichtern, die schlieszlich miteinander verschmelzen.

Der Eileiter der Knorpelganoiden entwickelt sich also aus den Trichtern der sekundären Nephrostomalkanälchen.

Ein Zusammenhang des Eileiters mit den Baumannschen Kapseln (N_2) läszt sich nur für die Anfangsstadien feststellen, später aber fallen die sekundären Nephrostomalkanälchen auseinander.

Unsere Untersuchungen haben uns zu folgenden Ergebnissen geführt: es sind zwei Typen von Eileitern zu unterscheiden: pronephrische Eileiter einerseits, mesonephrische andererseits. Der mesonephrische Typus zerfällt seinerseits in zwei Gruppen: Eileiter die sich aus sekundären Nephrostomaltrichtern entwickeln, und solche die aus primären Nephrostomaltrichtern gebildet werden.

I. *Selachia, Amphibia, Dipnoi und Amniota.*

Der Eileitertrichter entwickelt sich aus den Pronephrostrichtern, der Eileiterkanal aber aus einer Spaltung des primären Harnleiters in zwei Kanäle.

Die Samenleiter bilden sich aus dem primären Harnleiter, der Hodenzentralkanal — aus den primären Nephrostomaltrichtern.

II. *Knorpelganoiden.*

Der Eileiter entwickelt sich aus den sekundären Nephrostomaltrichtern des Mesonephros und mündet in das Ende des primären Harnleiters.

Der primäre Harnleiter dient zur Ausführung der Samen, der Hodenzentralkanal bildet sich aus den primären Nephrostomaltrichtern.

III. *Teleostei und Crossopterigii.*

Eileiter und Samenleiter stammen von den primären Nephrostomaltrichtern, und beide sind dem Hodenzentralkanal der beiden ersten Gruppen homolog."

As it will appear lower down, my results concerning the development of the oviduct in Acipenser ruthenus are absolutely different from those of MASCHKOWZEFF.

Concise exposition of the investigation.

For this inquiry I have made, after embedding in paraffin, series of transverse sections of $7\frac{1}{2}$—$10\,\mu$ thickness, which I stained with hematoxylin and eosin.

The microphotos have been taken by Mr. P. J. DE VRIES, instrument-maker 1st class at the laboratory.

Sterlet, 21 mm in length.

I did not succeed anymore than OSTROUMOFF in finding an oviduct in fishes up to 18 mm in length. However, my next-following specimen, 21 mm long, exhibits on the left side the ostium abdominale, just come forth, with the foundation of the duct, while these parts are still absent on the right side.

On photo 1 the ostium is observed under the middle of the section of the wolffian duct and laterad open to the body cavity. The lateral (dorsal) lip of the ostium leans right against that duct. The medial lip is formed by a laterally inclined fold of the somatopleura. This fold can still be followed as far as 17 sections towards the rostrum. It gradually becomes sagittal and smaller, while shifting medially to disappear under the medial margin of Wolff's duct.

Now let us follow the section of photo 1 candad; we shall then see on the next-following section (photo 2) that through the fusion of its two lips, the ostium is changed into a compact mass of cells, which can still be followed two sections further. The last of these two sections has been represented on photo 3. Here the conglomeration of cells is, as it were, hemmed in between the somatopleura and Wolff's duct. It is striking that the wall of the duct is two cells thick at the spot where it is in contact with the conglomeration, while for the rest it consists of a single layer of cells.

Sterlet 28 mm long.

In this stage of development the oviduct has appeared on either side. Also in this specimen the ostium abdominale is situated under the wolffian duct and laterad open to the body cavity. The lateral lip of the ostium appears as a slight swelling of the somatopleura towards the coelon. The medial lip is a laterally inclined fold of the somatopleura. This fold is present at about 25 sections rostrally from the closure of the ostium abdominale, which is engendered by fusion of the two lips. This fold reveals the same changes as the corresponding one of the sterlet of 21 mm. When tracing the sections caudad from the closure of the ostium abdominale the müllerian duct appears to contain a lumen at 75 sections.

Here follows a description of the successive sections of the end of the müllerian duct on the right side of the animal.

Photo 4. We see that the müllerian duct lies closer to the wolffian duct

than to the somatopleura. We also see that the wall of the müllerian duct, turned towards the wolffian duct, is richer in cells than the wall turned towards the somatopleura. Thirdly the wolffian duct is involute at the place of the müllerian duct.

Finally the wall of this involute portion of the wolffian duct is obviously thickened, it consists of several cell-layers, whereas the rest of its wall has only one cell-layer. The anomaly in its wall in the right top corner of the photo is due to the opening of a tubule of the mesonephros in the wolffian duct.

On the next section the lumen of the müllerian duct appears to have disappeared.

Photo 5. The boundary line between the blind end of the müllerian duct and the wolffian duct becomes vaguer. In the middle we see between them a cell, of which it is difficult to say whether it belongs to the one duct or the other.

Photo 6. This section presents a striking similarity to the schemata 477 and 478 on p. 506 of O. HERTWIG's Textbook.

In connection with the preceding photo it appears distinctly that the blind end of the müllerian duct goes on growing at the expense of the wolffian duct.

Photo 7. The involution of the wolffian duct has flattened. Of the cells in situ it cannot be said whether they belong to the caudal end of the müllerian duct, or to the wolffian duct.

On the next section the involution has flattened still more. For the rest it is like the preceding one.

Photo 8. The wall of the wolffian duct turned towards the coelom is now entirely parallel to the somatopleura. The thickening of the wolffian duct has disappeared.

We also examined fishes of 25 and 33 mm. Our experience with them agreed with that of the discussed sterlet of 28 mm. For the sake of brevity I omit a detailed discussion of these fishes.

Discussion and Conclusion.

MASCHKOWZEFF writes that the oviduct of the cartilaginous ganoids develops from the funnels of the secondary Nephrostomal-tubules. His argument is expressed in the above. He adds a reconstruction-scheme of a mesonephros-segment near the beginning of the müllerian duct in Acipenser, which I subjoin.

Beforehand I will observe that I borrowed the *literal* explanation of the figures from MASCHKOWZEFF.

Nowhere did I find in the larvae, cut by myself, the mesonephros derivates N_2. Neither in the sterlet of 21 mm, which assuredly presents an incipient stage of the development of Müller's duct.

Truly MASCHKOWZEFF adds:

"Ein Zusammenhang des Eileiters mit den Baumanschen Kapseln (N_2) läszt sich nur für die Anfangsstadien feststellen, später aber fallen die sekundären Nephrostomalkanälchen auseinander", but then at any rate segmental thickenings in the extremity of the müllerian duct should be observable and I never detected them in my preparations.

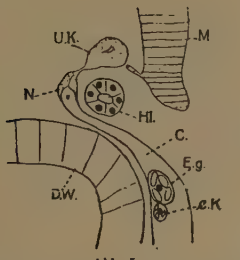


Abb. 5.

Abb. 5. Querschnitte durch den Embryo vom Accipenser Stellatus im Alter von 73 Stunden. *M.* Miotom. *Hl.* Harnleiter. *US.* Ursegmentstiel. *Dw.* Darmwand. *Eg.* große primäre Geschlechtszelle. *eé.* Kleine primäre Geschlechtszellen. *N.* Primäre Nephrostomaltrichter. *Ur.* Primäre Urnierenkanälchen. *C.* Colomhöhle.

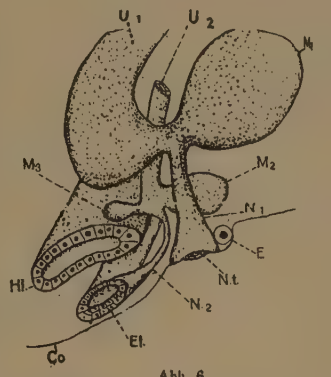


Abb. 6.

Abb. 6. Plastische Rekonstruktion eines Mesonephrossegmentes im Bereich der Eileiteranlage beim Accipenser Ruthenus. *El.* Eileiter. N_2 Sekundäre Nephrostomalkanälchen. M_2 Sekundäre Malpighische Körperchen. U_2 Sekundäre Urnierenkanälchen. M_3 Anlage der tertiären Malpighischen Körperchen. *Nt.* primäre Nephrostomaltrichter. *E* primäre Geschlechtszelle. N_1 primäre Nephrostomalkanälchen. M_1 primäre Malpighische Körperchen. U_1 primäre Urnierenkanälchen. *Co.* Colomepitellium.

I never found in any of the stages I examined the thickened stripe of peritoneal epithelium, which is a continuation of the medial fold at the beginning of the müllerian duct anteriorly, which OSTROUMOFF found in fishes of 35 mm and which he considers as a rudiment pointing to a phylogenetic relation to the posterior margin of the pronephros.

Contrary to OSTROUMOFF who, when speaking of the müllerian duct, says: "Welcher Kanal nach hinten in Gestalt einer soliden Anlage zwischen dem Peritonealepithel und dem Wolffschen Kanale weiterwächst", and who consequently assumes an independent growth of the solid extremity of the müllerian duct of Acipenser ruthenus, my investigations convinced me that the müllerian duct of Acipenser ruthenus grows under a supply of material from the wolffian duct. That OSTROUMOFF did not observe this mode of growth is quite comprehensible, because in his youngest specimen (35 mm long) the duct already extended over the same segments as in the full-grown animal that he examined (1½ m in length). In both cases he found the posterior margin of the ostium in the 22^{d} and the termination of the duct in the 27th segment. It is quite possible that the müllerian duct in a sterlet of 35 mm or more receives little or no material

from the wolffian duct, as it has already reached its terminal. Indeed, during the growth of the animal the duct gets longer just as the myotomes (length measured parallel to the body-axis), but this may be due to its own (already acquired) material, just as is the case with the myotomes.

In accordance with OSTROUMOFF I found in a sterlet of 78 mm that the müllerian duct extends over five segments. Its blind caudal end is, indeed, in direct contact with the wolffian duct, but this is no longer thickened and is not involute, so that it does not seem any longer to give off any more cells.

As known, the end of the müllerian duct in Acipenser lies at a considerable distance in front of the urogenital orifice. Referring to this OSTROUMOFF says of his full-grown specimen: "zwischen seinem hinteren Ende und dem unpaaren Genitalsinus liegen noch 10 Segmente". In the common terminal of the wolffian- and the müllerian duct still kidney-tubules are emptying, and this terminal to which also the "Genitalsinus" is to be counted, must of necessity be more than double the length of the müllerian duct.

It will not do, therefore, to say with MASCHKOWZEFF (see supra) about the Cartilaginous ganoids: "Der Eileiter mündet in *das Ende* des primären Harnleiters".

According to what has been said above, the pronephros duct in the sterlet has been split into two tubes only along 5 segments, and is, therefore, more primitive than in the Selachians, in which it is divided into two tubes over its whole length.

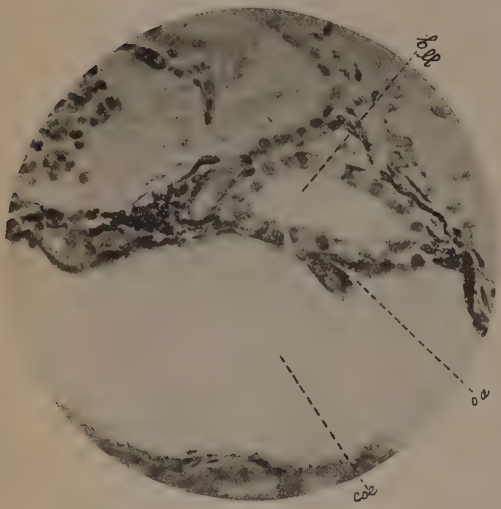
Also the front end of the pronephros duct (as far as the 22d segment according to OSTROUMOFF) remains unsplit in the sterlet, and in this respect it seems to agree rather with Amphibians and Amniotes than with the Selachians.

Summary.

1. Already at its first appearance the müllerian duct is in direct contact with the wolffian duct. This applies to both cases, as well when assuming as the beginning (front-end) of the müllerian duct the hind-end of the ostium abdominale (photo 2), as when considering as such the thickening in the wolffian duct, two sections further caudally. (Photo 3.)

2. The blind end of the müllerian duct, which goes on growing, keeps this contact also later on and in this place the thickening of the wolffian duct in young larvae is strong, while it consists everywhere else only of a single cell-layer.

3. The blind end of the müllerian duct is involved into this thickening and in younger specimens than the youngest (35 mm) examined by OSTROUMOFF the boundaries between the two ducts have entirely or partly disappeared. Here cells can be observed passing from the thickening to the müllerian duct (cf. photos 5, 6 and 7).



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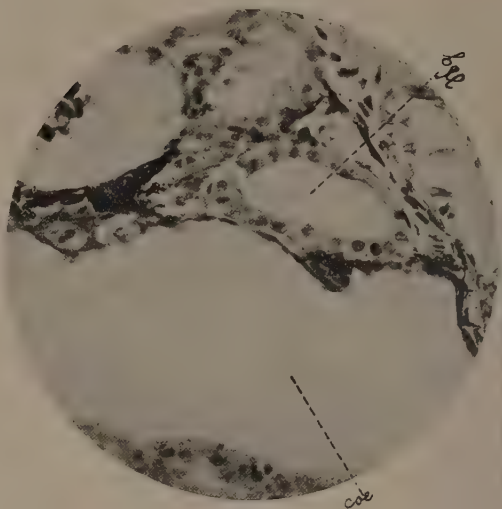

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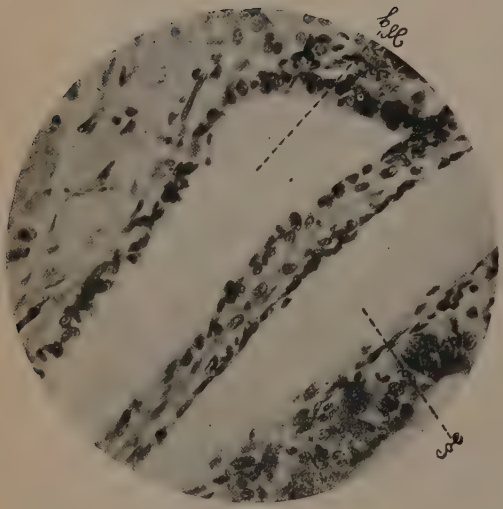

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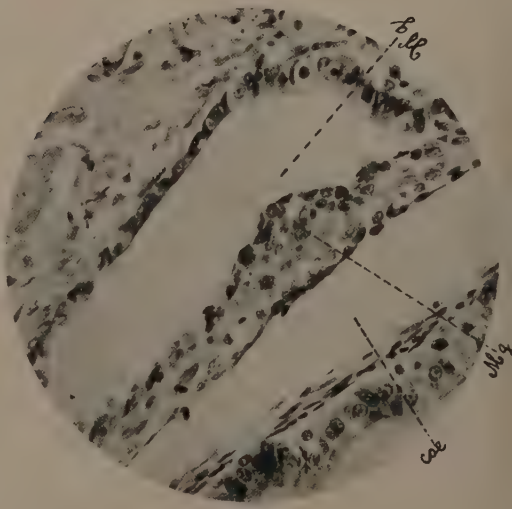

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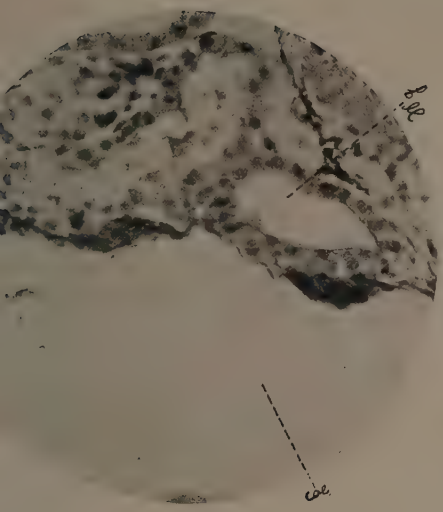

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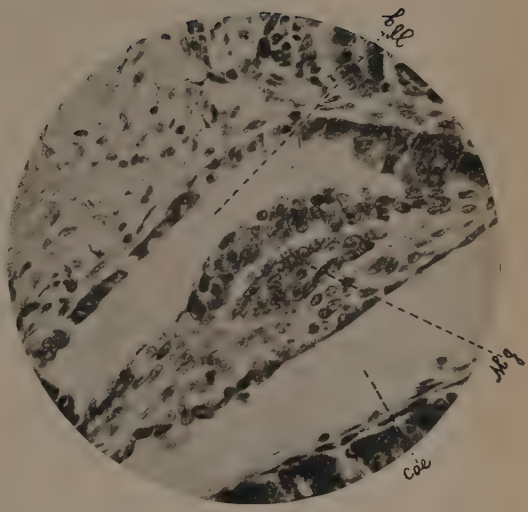

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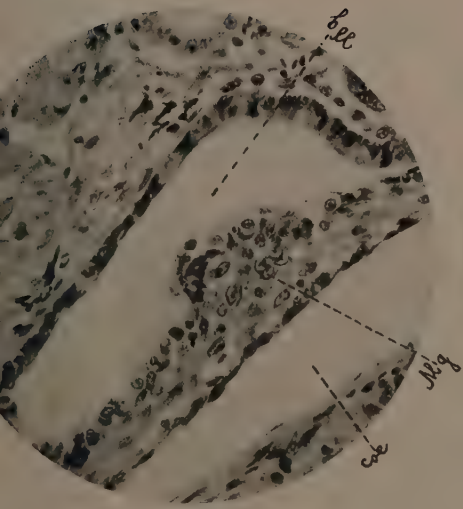

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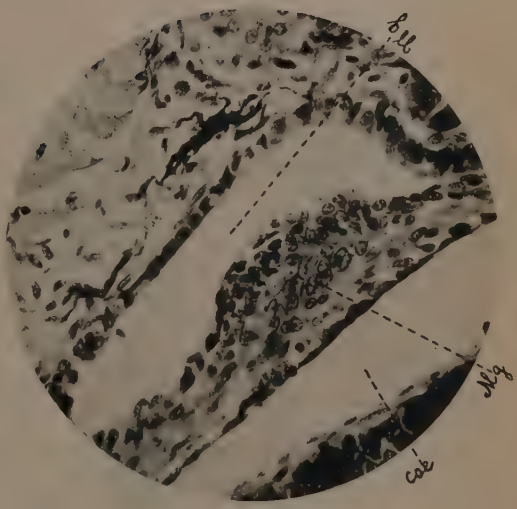

Foto 5.

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MASCHKOWZEFF, A. Entwicklung des Urogenitalsystems bei den Knorpelganoiden und einigen anderen Wirbeltieren. Anatomischer Anzeiger. Centralblatt für die gesamte wissenschaftliche Anatomie. Bd. **59**, 1924—1925.

OSTROUMOFF, A. (Kazan) Zur Entwicklungsgeschichte des Sterlets (Acipenser ruthenus). VI Der Müllersche Gang. Zool. Anzeiger, Bd. **33**, 1908.

EXPLANATION OF THE PLATE.

The photos 1, 2, 3 belong to a sterlet 21 mm long;
Fig. 2 photo first section after that of fig. 1;
Fig. 3 „ third „ „ „ „ „ 1:
The photos 4, 5, 6, 7, 8 belong to a sterlet 28 mm long;
Fig. 5 photo second section after that of photo 4;
„ 6 „ third „ „ „ „ „ 4;
„ 7 „ fourth „ „ „ „ „ 4;
„ 8 „ sixth „ „ „ „ „ 4;

Wg = wolffian duct; Mg = müllerian duct;
o a = ostium abdominale; coe = coelom.

Physics. — *The principal susceptibilities of Manganese Ammonium-sulphate crystals at low temperatures.* By L. C. JACKSON and W. J. DE HAAS. (Comm. Number 187c from the Physical Laboratory at Leiden.

(Communicated at the meeting of November 26, 1927).

Introduction. Crystalpowder of manganese ammoniumsulphate is known to follow the law of CURIE down to the lowest temperatures at which it has been examined [1]). The data available at the time of the research, seemed to indicate that the $1/\chi$. T-lines for the principal susceptibilities of paramagnetic crystals (if these lines are straight) are parallel to each other. It therefore seemed important to examine the principal susceptibilities of a crystal of manganese ammonium-sulphate. We might namely expect either the three principal susceptibilities to be equal and consequently the crystal to be magnetically isotropic, though belonging to the monocline system, or, what was more probable both positive and negative values of the constant $\triangle$ to occur in the formula $\chi(T+\triangle) = \text{constant}$.

It seemed however, just as well possible that each of the three principal susceptibilities follows the law of CURIE.

§ 1. *Experimental research.* We grew large well-formed crystals of manganese ammoniumsulphate and made sections from these in the shape of small cylinders with the aid of a crystalgrinding-goniometer, so that the axes of these cylinders form a definite known angle with the crystallografic axis. In order to determine the three principal susceptibilities and the angle between the susceptibility χ_1 and the c-axis three differently orientated disks from the crystal are necessary, while the fourth equation is given by the known value of the susceptibility $^1/_3\,(\chi_1+\chi_2+\chi_3)$.

The susceptibilities in the directions of the axes of the disks were then determined by measuring the force exercised on the small cylinders in a non-homogeneous field, that is with the method of CURIE. The value of $H\frac{dH}{dX}$ being not yet known at the place where the disk was put, the apparatus was calibrated with the aid of a thin celluloid capsule exactly of the same form as the little disks. It was fixed in the magnetic field in exactly the same position as the disks and filled with the finely divided material. The force on the empty capsule was proved to be neglectable.

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[1]) L. C. JACKSON and H. KAMERLINGH ONNES, Proc. Roy. Soc. London Oct. 1923. Comm. N°. 168a.

Thus the susceptibilities of the crystaldisks were determined in the temperatureregion of liquid hydrogen, boiling under different pressures. The apparatus used (somewhat changed for our purpose) has been described Comm. N°. 139*b*.

§ 2. *Method of calculation.* The axes of the three crystal disks were parallel to the „*b*"-axis, being the axis of symmetry of the crystal, perpendicular to the „*c*" (0 . 0 . 1) plane and perpendicular to the „*p*" (1 . 1 . 0) plane of the crystal respectively. When the susceptibilities in these directions are called *b*, *c* and *p*, respectively while *m* is the mean susceptibility, the main susceptibilities χ_1, χ_2, χ_3 (χ_3 coincides with the „*b*"-axis, χ_1 and χ_2 lie in the plane of symmetry) can be determined from the following equations:

$$\chi_3 = b \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$\chi_1 \cos^2\{\psi + (\beta - 90)\} + \chi_2 \sin^2\{\psi + (\beta - 90)\} = c \quad . \quad . \quad . \quad . \quad (2)$$

$$\chi_3 \cos^2\alpha + (\chi_1 \sin^2\psi + \chi_2 \cos^2\psi) \sin^2\alpha = p \quad . \quad . \quad . \quad . \quad (3)$$

$$\chi_1 + \chi_2 + \chi_3 = 3m \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where ψ is the angle between χ_1 and the „*c*" axis of the crystal and a the angle between the normal on the „*p*" plane ((1 . 1 . 0) plane) and the „*b*" axis, while β represents the angle between the „*a*" and „*c*" axes.

From these equations χ_1, χ_2, χ_3 and ψ can be solved.

§ 3. *Experimental results.* The following tables give the values of the susceptibilities as they have been observed.

Crystal disk "*b*"		Crystal disk "*c*"		Crystal disk "*p*"	
T	$\chi \cdot 10^6$	T	$\chi \cdot 10^6$	T	$\chi \cdot 10^6$
20°.3$_7$ K.	547	20°.3$_5$ K.	534	20°.3$_7$ K.	577
18.9$_4$	592	19.1$_4$	583	19.7$_7$	611
16.9$_4$	660	17.8$_2$	613	17.6$_0$	669
15.0$_7$	740	16.4$_0$	662	15.2$_2$	766
		15.0$_0$	738	14.8$_5$	791

In order to make the further calculation a set of values is necessary, which have been reduced to the same temperatures. Consequently graphs have been drawn of the values mentioned above of the χ's for the „*b*", „*c*" and „*p*" disks as functions of the temperature. The values of the „*b*", „*c*", „*p*" are then graphically reduced to $T = 20°.35$, 19°.0, 17°.0 and 15°.0 K.

These values follow here:

	Crystal disk			mean susceptibility of the powder „m"
	„b"	„c"	„p"	
T.K.	$\chi \cdot 10^6$			
20°.3$_5$K.	548	534	578	542
19.0	589	572	620	580
17.0	662	639	690	649
15.0	748	723	780	735

Moreover the following data [1]) are still required:

$$a : b : c = 0.7360 : 1 : 0.4972$$

$$\beta = 107°2'$$

$$p\,(1\,.\,1\,.\,0) : p\,(1\,.\,\bar{1}\,.\,0) = 70°16'$$

The final results of the calculation are:

T	$\cos^2\psi$		
20°.3$_5$K.	0.8518	of	0.1471
19.0	0.7813	„	0.2176
17.0	0.7936	„	0.2063
15.0	0.8366	„	0.1663

T	$\chi_1 \cdot 10^6$	$\chi_2 \cdot 10^6$	$\chi_3 \cdot 10^6$
20°.3$_5$K.	754	323	548
19.0	819	333	589
17.0	926	359	662
15.0	1038	420	748

The values in the third column (0.1471 etc.) have been accepted provisionally as the exact ones, as they come nearer to the values, which are known for the other members of the family of the monoclinic double

[1]) See GROTH, Chemische Kristallographie.

sulphates, to which manganese ammoniumsulphate belongs. No great accuracy may be expected however in the determination of ψ. The values of the principal susceptibilities will be accurate to about two percent. From these values we calculated the molecular susceptibilities, taking into account the diamagnetic property of the anion, the crystallization water and the ammoniumsulphate. The results are given in the following table. The values for χ'_{3m} are those, which have been observed with the aid of the crystaldisk „*b*".

T	χ'_{1m}	χ'_{2m}	T	χ'_{3m}
20°.3$_5$ K.	0.295	0.127	20°.3$_7$ K.	0.214
19.0	0.321	0.130	18.9$_4$	0.232
17.0	0.362	0.140	16.9$_4$	0.258
15.0	0.406	0.165	15.0$_7$	0.290

§ 4. *Results.* The inverse values of the corrected molecules susceptibilities are plotted against the absolute temperature. In this way the variation of the different susceptibilities with T has been represented

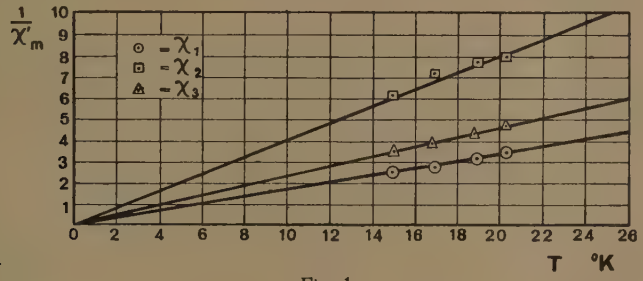

Fig. 1.

graphically. We see from Fig. 1 that the $\frac{1}{\chi}$. T lines are neither coïnciding, nor parallel. The values are lying as well as possible on three straight lines, which all pass through the origin.

This means that the principal susceptibilities follow the law of CURIE $\psi \,.\, T = C$, but with different values for the C.

The mean susceptibility of the crystalpowder of manganese ammoniumsulphate obeys the law of CURIE, because each of the principal susceptibilities follows this law.

Physics. — *Research about the question whether grey tin becomes supraconductive or not.* By W. J. DE HAAS, G. J. SIZOO and J. VOOGD. (Communication N°. 187*d* from the Physical Laboratory at Leiden.)

(Communicated at the meeting of December 17, 1927).

Introduction. The physical condition of a metal is known to have a great influence up on the conductivity. Also the form of the cristal-lattice has a predominant influence on different physical properties of the metals.

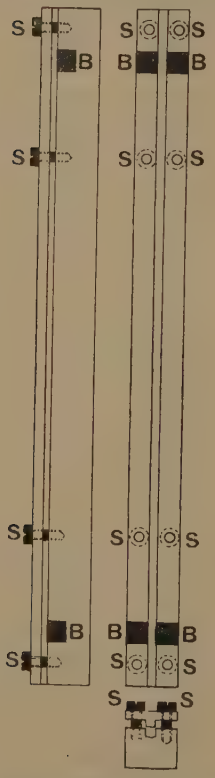

Fig. 1.

This induced us to investigate, whether grey tin, in the region of the temperatures obtainable with liquid helium behaved in a different way as white tin which, as we know, becomes supraconductive at about 3.6° K.

A difficulty for the research of grey tin is that it can only be obtained as a rough powder. The size of the grains of this powder ranges from 0.5 to 0.1 mm.

It was kindly made for us at the Van 't Hoff Laboratory at Utrecht by Professor COHEN, later during the latter's stay in America by Professor KRUYT. We heartily thank both gentlemen.

§ 1. The first experiments were made with a quantity of tin-powder pressed into a barshape. For that purpose a small piece of ivory was made with a groove in the length-direction of 1 mm width and depth. At some distance from the extremities four amalgamated brass pieces B (see fig.) were fixed to which the wires for the resistance-measurements were soldered. The groove was filled with grey tin and a lid, provided with a ledge exactly fitting into the groove, was screwed to the ivory by means of the screws. As it has no sense to measure accurately the resistance of such a badly defined resistance body and as moreover this was not our intention, the measurements were made with a plain Kohlrausch bridge. The resistances of the feeding wires were measured separately, so that a correction could be made for them.

The results of this research are given in Table I.

TABLE I.

Temperature	
300° K.	112.8
4.2	38.3
3.8	38.3
3.6	34.3
1.2	34.3

We can readily criticize this experiment. The current density becomes very great at the points of contact of the different metal grains and consequently a number of real resistances might arise either by heat-development or, what is more probable, by a too large magnetic field of the current itself, which, as is known, can disturb the supraconductive state. Also the grains might change their positions during the cooling, so that it would be preferable to press the ivory ledge into the groove with strong springs.

Anyhow, it would be desirable to repeat the experiment in a different way.

§ 2. With the second method we followed a suggestion of Prof. P. EHRENFEST and made use of the persistent currents in analogy with a method for gallium used by TUYN and KAMERLINGH ONNES.

A bar of grey tin was made inside a glass tube long 12 cm and with a diameter of 7 mm. An external coaxial field of 400 to 500 Gauss was excited and made to disappear. At its maximum strength such a field most probably was strong enough to destroy the supraconductive state of the metals; the supraconductive property of tin disappears namely between the temperatures 3.6° K. and 1.5° K. with fields ranging from 0 to 230 Gauss, for indium these values are from 3°.4 K. to 2°.3 K.; the fields from 0 to 130 Gauss, for mercury from 4°.2 K. to 1°.87 K., the fields from 0 to 340 Gauss, for lead from 7°.2 K. to 3°.7 K. and the fields from 0 to 600 Gauss.

If now we let the maximal field disappear we cross the eventual maximal magnet temperature threshold value of the metal and rouse the persistent currents in every grain. At the extremities of the bar of grey tin outside the cryostate an easily revolving magnet has been placed to discover the existence of the persistent currents. The experiments were made at the boiling point-pressures mentioned below and at the temperatures of liquid helium.

TABLE II.

Boiling point of Helium.	Temperature
774 mm.	4.21° K.
350	3.50
165	3.00
70	2.55
45	2.35
40	2.30
36	2.26
33	2.23
27	2.15
20	2.04
10	1.80

§ 3. *Conclusions.* No declinations of the magnetic needle were stated. We may conclude from this that grey tin does become supraconductive even at the lowest temperatures used. We might still object that the grey tin might have been in the supraconductive state continually, also at the highest temperatures and in the highest fields used. A rise from 0 to 500 Gauss and a vanishing again of the fields should have had no influence in that case. However, this supposition is hardly probable with a view to the experiments under § 1. And yet, in order to get absolute certainty we have repeated the experiment with the bar of grey tin in such a way that we excited the coaxial magnetfield, before the cryostate with the bar of grey tin had been cooled. Only after the gradual cooling of the Helium-cryostate-glass and the reducing of the boiling-point of the Helium to the low temperature of 1.8° K., the magnetic field was made to vanish. This time too the bar of grey tin did not show any persistent currents.

While KAMERLINGH ONNES [1]) came to the conclusion that a loose atom connection and a great distance between the atoms were to advantageous for the creation of the supraconductive state the above experiment shows that the phenomenon is more complicated.

Grey tin has a smaller density than white tin.

[1]) Leiden. Comm. Suppl. N°. 50.

Physics. — *On the change of colour of crystals at low temperatures.* By I. OBREIMOW and W. J. DE HAAS. (Comm. N°. 191a from the Physical Laboratory Leiden.)

(Communicated at the meeting of January 28, 1928).

Introduction. In contrast with the spectra of gases, the absorption spectrum of solid bodies and liquids consists as a rule of very large, sometimes very diffuse bands. But in the last few years more and more experiments have shown that, by sufficiently lowering the temperature, the crystalspectra are transformed from large bands into sharp spectral-lines. 1906 already JEAN BECQUEREL observed, that the crystals of the rare earths, when immersed in liquid air, gave absorption spectra, which consisted of a large number of very fine bands [1]).

It must be mentioned that already in some cases of the absorption-spectra, the spectral terms have been found and that the classification of the lines has succeeded [2]).

The well-known change of colour [3]), which appears at the cooling of crystals, suggests the formation of absorption-bands and a change in the structure of the bands.

The purpose of this research is to examine, whether

1°. the change of colour, which has been observed down to the temperature of liquid air continues in the temperature-region of liquid hydrogen and

2°. whether in this case the absorption-bands are perhaps transformed into spectral lines.

§ 1. Three different crystals were subjected to a preliminary research viz. the crystals of azobenzol, potassium bichromate and iodine. All three crystals change their colours at low temperatures. Azobenzol and potassium bichromate become citrine-yellow and iodine dark red.

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[1]) J. BECQUEREL, Le Radium, **4**, 328, 1907. See for further literature among others.
V. HERI, Proceedings of the Optical Institute I, 2.
M. DE SÉLINCOURT, Proc. R.S. **107**, p. 247, 1925.
L. VEGARD, Comm. Number 175, Leiden.
RUBENS und G. HERTZ, Stzb. d. Preuss. Akad. I, p. 256, 1912.
A. KRONENBERGER und P. PRINGSHEIM, Z. f. Phys. **40**, p. 75, 1926.
B. GUDDEN und R. POHL, Ph. Zs. p. 481, 1925.

[2]) A. G. S. VAN HEEL, Dissertation 1925. See for the theory P. EHRENFEST, Memorial Volume H. KAMERLINGH ONNES, Leiden 1922.
See for literature also P. PRINGSHEIM, Fluorescenz und Phosphorescenz im Lichte der Neueren Atomtheorie.

[3]) See among others KOURBATOFF, Journ. Russ. Chem. Ges. **39**, II, p. 134, 1907.

The method of investigation was exceedingly simple and hardly wants a detailed explanation.

The light of an arc-lamp is cast through the crystal plates. After having first been carefully cooled by the vapour of liquid hydrogen, these are immersed in the liquid hydrogen boiling under atmospheric pressure.

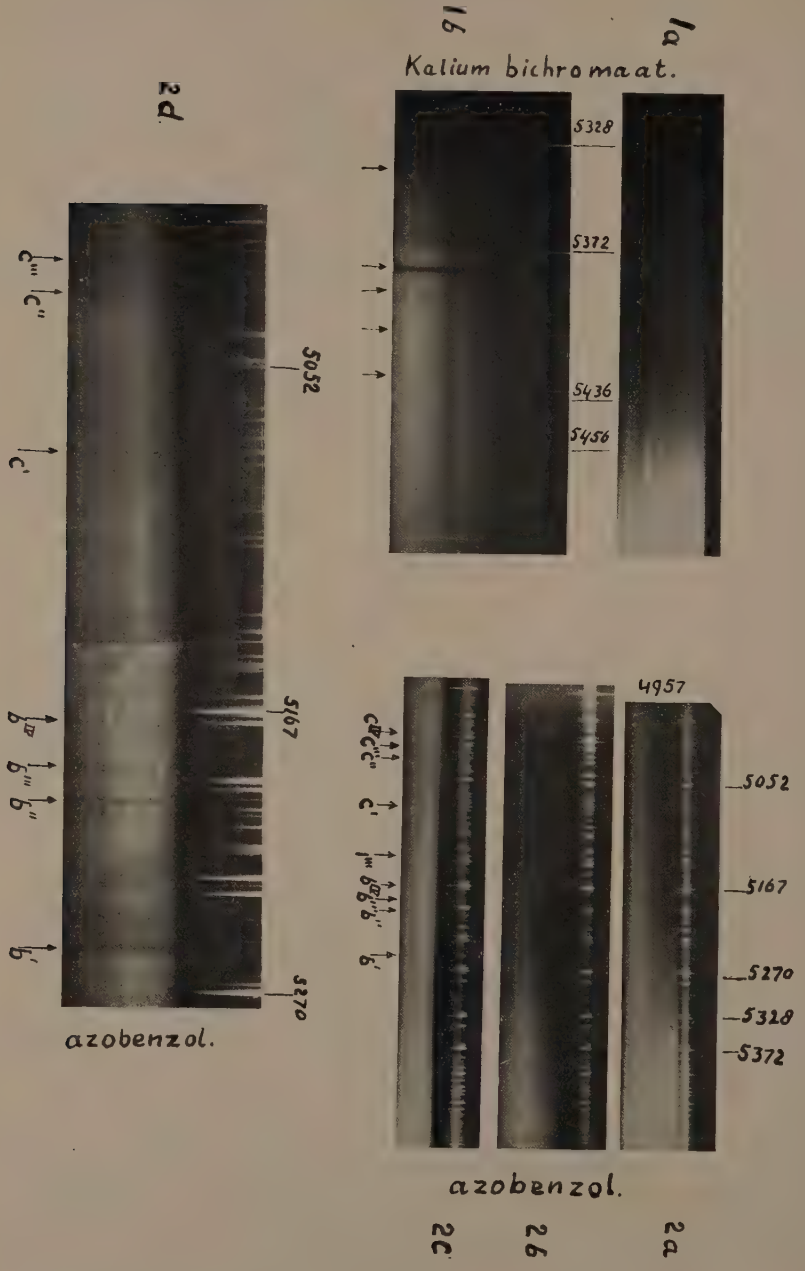
The vacuum-vessel in which the hydrogen boiled, was made of quartz. Small windows of plane quartz glass had been fused in the walls to let the light pass, perpendicularly to the axis of the vessel.

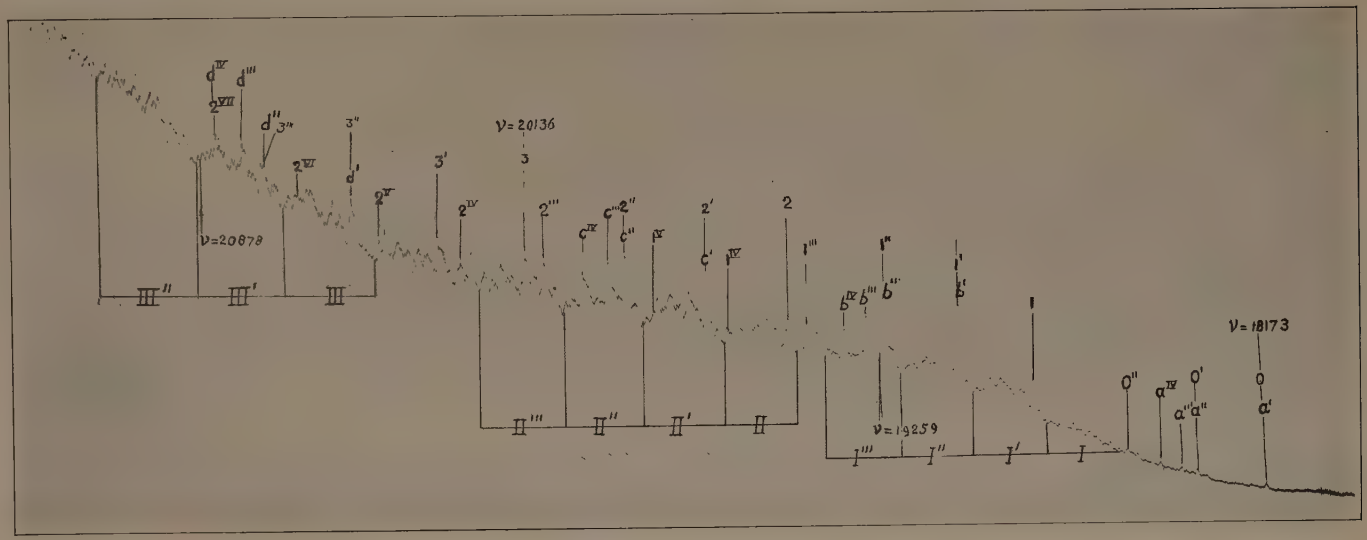
The light through the first two quartz windows, falls on the crystal which, with the aid of cork is fixed by mean of small springs in a holder, passing through the cryostatecap. The light after passing through the crystals leaves the vessel by two quartz windows and falls on the slit of a Steinheil spectrograph. The dispersion of this instrument is such that the *D*-lines are separated over a distance of about $^1/_3$ mm. For the determination of the wave-lenghts of the absorption-lines the spectrum of an iron arch is cast at the same time on the photographical plate. On account of their large absorption the crystals have sometimes been thinly cut (for azobenzol about 0.1 mm, for potassium bichromate about 0.2 to 0.3 mm and for iodine also about 0.2 to 0.3 mm). Their dimensions were prescribed by these of the opening in the cork holders 2 to 3 mm).

§ 2. *Potassium-bichromate and iodine.* At room temperature potassium bichromate has a continuous absorption-band, which begins at nearly 5500 Å and extends into the extreme ultraviolet (fig. 1*a*). At 20° T.K. the red side of the absorption-band is split up into some sharp lines (see fig. 1*b*). The potassium bichromate shows a distinct pleochroism; when we place a nichol in the beam of rays of the crystal, the spectra change with the azimuth of the nichol. Fig. 1*b* shows the enlarged spectrum of a potassium bichromate crystal (0.2 mm thick) taken with an incident polarized beam of rays. The orientation of the crystal had not been determined. We intend to repeat these experiments and give here only this preliminary communication.

Iodine-sheets (0.1 mm thick) are not transparent at ordinary and liquid air temperatures. These same sheets at 20° T.K. (liquid hydrogen) however, transmit dark red light (> 6700 Å). In the transmitted light we see very weak, periodical bands. Pleochroism may be very distinctly observed. The experiments with iodine have not been continued for the present, as the sheets have to be made thinner and have to be studied at still lower temperatures.

§ 3. Azobenzol, $C_6H_5 - N = N - C_6H_5$ crystallizes in the monocline system ($a : b : c = 2.11 : 1 : 1.33$; angle $(a, c) = 114°26'$). From an alcoholic solution we can precipitate the crystals in the shape of thin sheets by evaporation. The base of these sheets is c (001). Plane of the axes $\perp$ to the plane of symmetry (010). When we illuminate these sheets by white


1b
1a
Kalium bichromaat.
5328
5372
5436
5456
2d
c'''
c''
c'
5052
5167
b'''
b''
b'
5270
azobenzol.
4957
5052
5167
5270
5328
5372
azobenzol.
2c
2b
2a


ν = 20136
ν = 20878
ν = 19259
ν = 18173

light they are very pleochroistic. They show an orange-colour, if the electric lightvector is parallel to the *b*-axis and a citrine-colour, if the lightvector is perpendicular to the direction mentioned above. At a temperature of —180° C one component becomes of a citrine-yellow colour and the other of an almost white very light green colour. The spectroscopical research shows that in the case of the second component, the plate is transparent for the whole visible spectrum.

We examined the „red" component only. Fig. 2*a* shows the „red" absorption-spectrum at room temperature. It doesn 't show any structure. Fig. 2*b* shows the same absorption spectrum at —196 C (liquid nitrogen). The edge of the very large absorption-band breaks up into a series of narrower but still large and diffuse bands. About 13 bands could be counted; in the fig they are not very well visible; in the blue the absorption becomes too strong to be able to observe still anything. At a temperature of 20° T.K. (liquid hydrogen) azobenzol shows a splendid absorption-line-spectrum (fig. 2*c*). In the case of azobenzol, as well as in the other cases described above, the change of colour continues because of the narrowing of the absorption-bands. In fig. 2 a small part of a diagram of azobenzol at usual temperature and one at 20° K.T. have been reproduced above each other. We see from this fig. 2*b*, 2*c* that the absorption-bands are not perceptibly shifted, but that the bands are split up into lines. In the reproduction the pictures lose considerably in contrast. Moreover the same part of the absorption-spectrum has been represented 10 × enlarged in fig. 2*d*.

§ 4. We cannot yet communicate the classification of the lines and we have the intention to examine the symptoms partly at still lower temperatures. Yet some regularities and periodicities are so striking that we will mention them here.

The periodicity is very striking for azobenzol. We have taken a photogram from the spectrum of the azobenzol represented in fig. 3. We can state distinct periodical variations of the intensities indicated by the figures I, I′, I″, I‴, II, II′, II″, II‴, III, III′, III″, etc. The medium width of the periodicity I is 17 mm, of II 18 mm of III 21 mm.

We want to draw the attention to the group of lines marked *a′*, *a″*, *a‴*, *a⁗* ; *b′*, *b″*, *b‴*, *b⁗* ; *c* etc., *d* etc. This group of 4 lines repeats itself four times. The distances between the lines of a group are given below:

	a	*A*	*b*	*B*	*c*	*C*	*d*	*D*	mean
I	0	0	0	0	0	0	0	0	0
II	15.8	1	17.2	1	18.7	1	20.9	1	1
III	19.3	1.22	21.1	1.23	22.4	1.20	25.2	1.20	1.21
IV	24.1	1.52	26.2	1.53	28.1	1.55	31.3	1.50	1.51

In this table the distances *a′*—*a″*, *b′*—*b″* etc. have been taken as a unit

for the vertical columns a, b, etc. and from these the numbers A, B, etc. have been calculated.

The microphotogram (fig. 3) shows that we have succeeded in splitting up the band into isolated lines as is for instance evident for the lines a', a'', a''', a'''', b', b'', b''', b'''', c', c''. But towards the violet end of the spectrum the absorption lines are lying so closely together that it is difficult to find the real place of their centre.

For, if

$$y_1 = f_1(\nu) \quad y_2 = f_2(\nu)$$

represents the distribution of the absorbed light in two isolated lines and if both coïncide, the light absorption caused by both the lines, will be represented by

$$y = y_1 + y_2 = f_1(\nu) + f_2(\nu).$$

The maximum absorption will lie at

$$y'_1 + y'_2 = 0,$$

viz there where the slopes of both absorption-curves are equal and opposite, but not at the values of ν where

$$y'_1 = 0 \text{ or } y'_2 = 0.$$

As however these latter values of ν give the centres of the lines it is desirable to fix only the place of the good undisturbed lines. The fixing of weak lines may lead to a mistaking of the place of the maximum absorption for that of the centre of a single line.

Consequently we have not fixed the place of all maxima, but it seems that the total spectrum is a periodical repetition of the same line-groups. It seems that the lines marked as 1, 1′, 1″, 1‴, possess the same distance in frequency.

This is also the case with the lines 2, 2′, 2″, 2‴, 2⁗, 2⁗′, 2⁗″, and with the lines 3, 3′, 3″, 3‴, and with the lines 0, 0′ and 0″.

The experiments will be continued with some larger dispersion of the spectrograph and if necessary at lower temperature.

Finally we kind thank Mr. P. M. VAN ALPHEN, who made the microphotograms for us and Mr. G. J. FLIM for his valuable help with the construction of the apparatus.

Ophthalmology. — *On the Analysis of Ocular movements.* By C. D. VERRIJP. (Communicated by Prof. J. VAN DER HOEVE.)

(Communicated at the meeting of February 25, 1928).

As the spatial position of an object is known from the location of three fixed points on the object, the analysis of the eye-movements may be imagined to be effected through the localization in a number of successive positions of three points indicated or supposed on the eye. If the assumption be admitted that the eye can perform such movements that all points of it describe paths parallel to some level plane (e.g. the horizontal), it will suffice for the analysis of *these* movements to localize two fixed points. We are justified in assuming that, at least for the horizontal and the vertical plane, such movements are possible. I will confine myself to this kind and first of all consider in particular those movements in which all points describe paths parallel to a horizontal plane.

Now, if of two fixed points the place is known for a number of successive positions, we can determine how the eye has shifted, respectively has rotated, and that with a precision that depends besides on the accuracy of the observations also on the minuteness of the displacements about which observations are made. As, in the experiment it is not possible to accurately establish the place of the fixed points indicated or supposed on the eye, I have relinquished this method. The object in view may also be attained if with every position a line fixed with respect to the eye, and one fixed point are localized. The visual line [1]) I take for fixed line, for

[1]) The current nomenclature distinguishes (HELMHOLTZ's Handb. der physiol. Optik:) 1^0 the "Blicklinie", i.e. the straight line, drawn from the point of fixation to the centre of rotation of the eye: 2^0 the "Gesichtslinie" (visual line) consisting of two parts, the front-portion of which is a straight line, joining the fixation-point to the first nodal point, the posterior portion being the straight line running from the second nodal point to the fovea centralis. Besides these lines also "Visierlinie" was spoken of, i.e. a line drawn through two points covering each other, which means that the point is situated in the middle of the dispersed image of the other; since the cone of rays, which forms the dispersed image, is limited by the opening of the pupil, these "Visierlinien" (lines of sight) should intersect each other in the centre of the pupil. In the third edition of HELMHOLTZ's manual GULLSTRAND observes that, since for "visieren" (aiming, directing) central vision is required, only one line of sight can be concerned, which after the refraction touches the central point of the fovea centralis. In connection with the fact that it is not likely that we may speak of one single centre of rotation, I call the visual line ("Blicklinie") the straight line which in the normal, not squinting eye joins the point of fixation with the middle of the fovea centralis, the place of the highest visual acuity.

fixed point the intersection of this line and the anterior surface of the cornea.

I purpose to describe in a following paper the experiments by which this is realized.

The problem is now reduced to the analysis of the displacement of a portion of a straight line in a level plane.

This level plane is the horizontal plane in which the visual line is moving. The experimental data are a number of successive positions of this line and the spots, (expressed in co-ordinates) which the intersection of the anterior surface of the cornea with the visual line, occupies every time.

When a portion of a line moves in a flat plane from a position AB to a position $A'B'$, a point P can *always* be indicated that is equidistant from all corresponding points of AB and $A'B'$ (the intersection of two lines which divide vertically in two the connecting lines of two pairs of corresponding points, as may easily be proved). The portion AB may then have come in $A'B'$ through a rotation in which all the points have described arcs around the centre P. It is clear, however, that *in general* this rotation may be assumed only when the described angle of rotation is infinitely small. If this should not be so, the displacement from AB to $A'B'$ may also be conceived as originating from one or more shifts of AB parallel to itself, combined with one or more rotations, whereby the place of the centres of rotation and the extent of the shiftings *remain in that case absolutely undetermined*. However, the movement may always be considered as the succession of infinitely small rotations, every following one round another centre of rotation, now *altogether determined*; the object of the analysis is to find the line described by this ever varying centre of rotation.

The centres of rotation thus found are properly speaking the intersections of the *axes*, round which the eye rotates, with the flat plane, in which the visual line is moving.

But apart from this it is also evident that, in connection with the foregoing, we should not a priori speak of one single centre of rotation, without knowing the movement. Every experimental method that *starts* from the hypothesis that there is only one centre of rotation, is objectionable. This is the more cogent as i.a. on anatomic grounds we are induced to doubt a simple rotation round one single point.

It should be required of the experimental method that it shows us the place occupied by the eye in a number of successive positions. The analysis of these data furnishes the curve of the instantaneous centres of rotation.

When looking up the literature to see how the various investigators have conceived the problem, it seems to me that they have, generally speaking, not formed a correct kinematic idea about it. When we determine the lines of sight for the different positions of the eye, and we find that they intersect in one and the same point, it is clear that this point need not be the centre of rotation of the eye. Already BERLIN pointed to this fact. Conversely, when we find that these lines do not intersect in one point, it is not admissible, to conclude from it that there has not been one

centre of rotation. But by the methods in which *"the"* centre of rotation is not assumed beforehand on a definite line, we do not attain our purpose either. At best they serve to prove that *the* centre of rotation does not exist. If I am not mistaken, only one method has been described that meets the requirements. It is that of FICK, which J. J. MÜLLER [1]) used in an investigation. He gives the place of the line of sight and that of the intersection of this line with the anterior surface of the cornea for a number of successive positions of the eye, each of them forming a rather small angle (about 5°—7°) with the following one. From the fact that the named intersections lie approximately on a circle, he concludes that there is only one centre of rotation, viz the centre of this circle.

This conclusion, however, would be justified only if the lines of sight were tangents of a circle with the same centre (or if they intersected in the same centre). But since the author states that this is most often not the case, it seems to me that the results of this inquiry go rather against the hypothesis of one single centre of rotation.

I shall not dwell further on the literature [2]), but will only enter into a speculation of the method described by KOSTER [3]).

True, this method is founded originally on the assumption of one centre of rotation. However, the author has made complementary measurements, with which after all an image of the eye-movements can be projected.

In connection also with the fact that it is difficult to judge of the accuracy of the data already obtained, I thought it in every respect expedient to undertake new experiments, in which I should start directly from the above speculation.

As stated before, in every new position of the eye the direction of the visual line and the co-ordinates of a fixed point on that line are measured. The position is then established completely. The following considerations now lead to the construction of the curve of the centre of rotation (see fig. 1).

In the figure let the X-axis be positive downward and the Y-axis positive to the right. Let the zero-position of the visual line coincide with the X-axis; let the fixed point A° on it be in that zero-position in the intersection of X-, and Y-axis; now the eye rotates and the visual line takes up e.g. a position $A'B'$. The angle described by the rotating eye we call α_n. The co-ordinates of the point A' we call in the direction of X-, and Y-axis respectively b_n and c_n. Now let us watch the movement, when the eye removes from the position $A'B'$ to the position $A''B''$ and let us thereby assume for a moment a rotation round one single point P. The co-ordinates of A'' are b_{n+1} and c_{n+1}; the angle over

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[1]) J. J. MÜLLER, Arch. f. Ophth. XIV Jahrgt, Abt. 3, p. 183; 1868.

[2]) A bibliography is given by BRENNECKE (Klin. Monatsbl. f. Augenh., Bd. 68 p. 227, 1922) who himself seems to have determined only the intersections of lines of sight; also by KOSTER, and by C. SCHAAP, Thesis. Leiden 1927.

[3]) W. KOSTER, Arch. Néerl. des sciences exactes et nat. T. XXX, p. 370. 1897.

which the eye has moved from the zero-position we call α_{n+1}. From $A'B'$ to $A''B''$ the angle of rotation was $A'PA'' = \alpha_{n+1} - \alpha_n$. This angle we name δ_n. Of course the angle between the visual lines is the same,

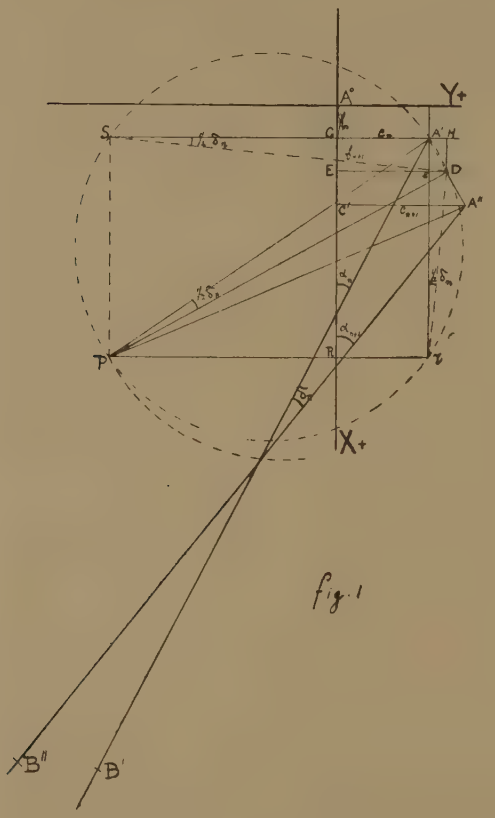

fig. 1

just as the angle between $A'B'$ and the X-axis is α_n. When drawing from A' the line $A'r$ parallel to the X-axis, and from P the line Pr normal to it, then the line PA' is a diameter of a circle passing through the points P, A' and r, because $\angle PrA' = 90°$; this circle cuts the line $A'A''$ in D and it will easily be seen that, as now also $\angle A'DP = 90°$, and consequently $\triangle PA'D \cong \triangle PA''D$,

$$A'D = A''D \text{ and } \angle A'PD = \angle A''PD = \tfrac{1}{2}\delta_n.$$

When producing the perpendicular $A'C$ as far as S and erecting the perpendiculars DE on the X-axis and DH on $A'C$, then it will be seen

that also the angles $A'SD$ and $A'rD = \frac{1}{2}\delta_n$. Let the co-ordinates of the centre of rotation P be called ξ and η then

$$\xi = A°R = A°E + ER = A°E + er = A°E + eD \cot \angle erD \text{ and}$$
$$\eta = CS = CH - SH = ED - HD \cot \angle HSD,$$

from which are derived the general formulae

$$\xi = \tfrac{1}{2}(b_{n+1} + b_n) + \tfrac{1}{2}(c_{n+1} - c_n) \cot \tfrac{1}{2}\delta_n \quad . \quad . \quad . \quad . \quad (1)$$

and

$$\eta = \tfrac{1}{2}(c_{n+1} + c_n) - \tfrac{1}{2}(b_{n+1} - b_n) \cot \tfrac{1}{2}\delta_n \quad . \quad . \quad . \quad . \quad (2)$$

So the location of the centre of rotation can be readily computed from the coordinates of A' and A'' and the angle of rotation, which is likewise the angle between the two visual lines.

This speculation was founded on the assumption that the displacement from $A'B'$ to $A''B''$ occurred through rotation round the point P; which assumption is admissible only if the angle between $A'B'$ and $A''B''$ is infinitely small. In this respect the above formulae may still be simplified. For first of all $\frac{1}{2}(b_{n+1} + b_n)$ draws near to b_n and $\frac{1}{2}(c_{n+1} + c_n)$ to c_n. The co-ordinates of A' are both functions of α. We write $b = F(\alpha)$ and $c = f(\alpha)$. It can now easily be seen that the last terms of the second sides of the above equations are nothing but the differential coefficients of these functions, for e.g.

$$\tfrac{1}{2}(c_{n+1} - c_n) \cot \tfrac{1}{2}\delta_n = \frac{c_{n+1} - c_n}{2 \times t_g \frac{1}{2}\delta_n} = \frac{c_{n+1} - c_n}{\delta_n} \to f'(\alpha),$$

when δ_n and α are expressed in radials. If α is expressed in degrees this expression becomes $\frac{180}{\pi} \times f'(\alpha) = 57.2958 \times f'(\alpha)$. The equations (1) and (2) change into the following:

$$\xi = F(\alpha) + 57{,}2958\, f'(\alpha) \quad . \quad (3)$$

and

$$\eta = f(\alpha) - 57{,}2958\, F'(\alpha) \quad . \quad (4)$$

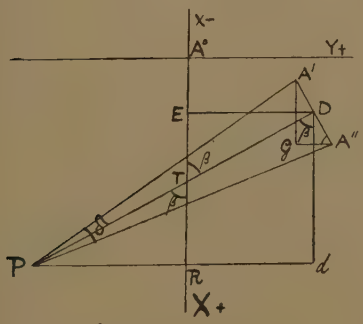

fig. 2.

These equations can also be derived directly (Fig. 2).

Here the distance $A'A''$ represents a very small displacement, the rotation taking place over an angle δ, round the centre of rotation P; PD we call ϱ, the angle this line makes with the X-axis β. Then is

$\beta = \angle DT A_0 = \angle PTR = \angle P Dd = \angle A' A'' G$. Further we have: $\xi = A^0R = A^0E + ER = A^0E + Dd$.

Now $A^0E = b + \frac{1}{2} \triangle b$, when expressing the very small increase of b : $A' G$ by $\triangle b$. Furthermore

$$Dd = \varrho \cos \beta = \varrho \frac{GA''}{A'A''} = \varrho \times \frac{\triangle c}{\varrho \delta}$$

(δ is expressed in radials and $A'A''$ may be looked upon as an arc): now

$$\frac{\triangle c}{\delta} = \frac{\triangle c}{\frac{\pi}{180} \triangle a} = 57{,}3 \times \frac{\triangle c}{\triangle a}$$

when $\triangle a$ represents, expressed in degrees, the increment of the angle which the visual line forms with its zero-position (direction of the X-axis); naturally this increment is of the same magnitude as $\angle A'PA''$. So we get:

$$\xi = b + \tfrac{1}{2} \triangle b + 57{,}3 \times \frac{\triangle c}{\triangle a}.$$

This expression has for limit, if $\triangle b$, $\triangle c$ and $\triangle a$ are infinitely small,

$$\xi = F(a) + 57{,}3 \times f'(a).$$

In the same way we can derive:

$$\eta = RP = Rd + dP = ED + dP = c + \tfrac{1}{2} \triangle c - \varrho \sin \beta = c + \tfrac{1}{2} \triangle c -$$

$$- \varrho \times \frac{\triangle b}{\varrho \delta} = c + \tfrac{1}{2} \triangle c - \frac{\triangle b}{\frac{\pi}{180} \times \triangle a},$$

the limit of which expression is again:

$$\eta = f(a) - 57{,}3 \times F'(a).$$

It is evident that only the formulae (3) and (4) are theoretically quite correct. If for a definite point of the curve, described by A, we wish to establish the corresponding centre of rotation (corresponding with the small piece of the curve of which A (or D) is the middle) we should try to construct $F(a)$ and $f(a)$ from the experimental data about b and c, after which ξ and η can be computed for the said point A. But then it is necessary, we should have very accurate data concerning the curve, described by A: every slight deviation in this curve may bring on great changes in the place of the centre of rotation.

Meanwhile with the aid of the formulae (1) and (2) an image may be projected that, especially when making an adequate number of observations, need not differ much from reality.

The difference with the second method, which uses (3) and (4) is, that the first method considers only two points of the A curve, not the points between them nor those round about them, and two visual lines, whereas the secound method gives a speculation *in connection with the surrounding points*; this is quite rational owing to the fact that the curve must show some gradation, that is to say that the location of all the points affects in any case that of the neighbouring points. But errors in the observations will count all the more now. With the second method, not with the first, the exact condition might be derived from correct observations. Yet with observations, not perfectly correct, there is every chance to project an image with the first method, which is more like the real image than the one obtained by the second method. I shall have an opportunity to demonstrate this when discussing the data furnished by KOSTER's observations.

Discussion of KOSTER's *method.*

This method was published as early as 1897. Strange to say, the notion of one centre of rotation has continued in the literature, in spite of KOSTER's results, which give evidence to the contrary.

The method starts from the assumption of one centre of rotation. Three positions of the eye are considered, a central position, in which the visual line Mo (fig. 3) runs straight forward, and to the left and to the right a

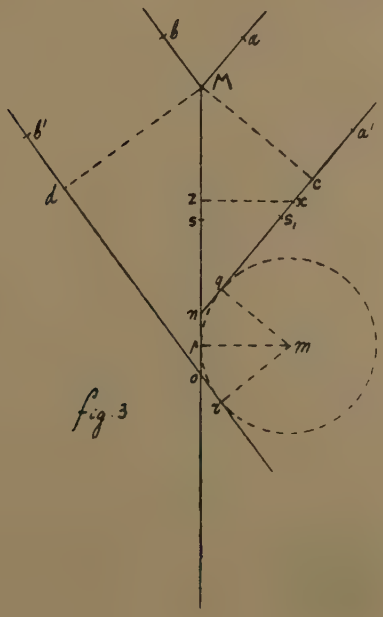

fig. 3

lateral-position, of which the visual lines *do* and *cn* are at the same angle α with the central position. This is realized in the experiment in the following way: on a graduated arc of which the centre is in *M*, a movable sector is mounted, so that with every movement it rotates round *M*. To this sector is applied a small aiming-telescope for fixation that can be shifted parallel to itself, as well as in the direction of the axis, which shiftings are measurable. Now it is first arranged so as to make the visual line fall along *Mo*. Subsequently the telescope is moved over an angle α, measured on the protractor, first to the one side and then to the other, every time determining how far it has to be shifted parallel to itself (*M c*, respectively *M d*) to render fixation possible again. The direction of the visual lines is then indicated *q c*, respectively by *r d*. If now it might be assumed that the eye had rotated continually round the one point *m*, the place of point *m* could be derived from the data obtained, for a simple computation shows that

$$Mp = \frac{Mc + Md}{2 \sin \alpha} \quad \text{and} \quad pm = \frac{Md - Mc}{2 \sin \alpha \, tg \, \frac{1}{2} \alpha}.$$ [1])

So, if there is one centre of rotation, its place can, indeed, be determined by help of this method. Additionally by also measuring *M s:* the distance from *M* to the intersection of the anterior surface of the cornea with the visual line in the middle-position, the place of the centre of rotation in respect to that intersection can be indicated. If, however, different values are found for *M p* and *p m* for different values of α, it follows that we cannot speak any longer of one centre of rotation. On the other hand, we are not sure that, if the same values are always found for *M p* and *m p*, there is really one centre of rotation. For the only thing we have established, is the position of the lines along which three visual lines are running, of which the middle one makes equal angles α with the other two: this does not at all determine the piece of the line the eye cuts off in the three different positions. Suppose *all* visual lines should touch the circle with *m p* for radius, so that the same point *m* would always be found, it would be possible that, without one's noticing it, the distance from one and the same point, e.g. *s*, of the eye to this point *m* always varies. If such be the case, there is of course no question about one single *centre of rotation*. KOSTER found different points *m*; from this it follows that there is not one centre of rotation. But besides this *the eye has never rotated round one of those points m*. This could be the case only if three positions were con-

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[1]) If considering one direction of vision straight forward and two side-positions on *the same* side the one forming an angle α, the other an angle 2α with the first, we get:

$$Mp = \frac{2 \, Mc \, (1 + \cos \alpha) - Md}{2 \sin \alpha} \quad \text{and} \quad pm = \frac{2 \, Mc \cos \alpha - Md}{2 \, (1 - \cos \alpha)}$$

sidered, of which the middle one would make a very (infinitely) small angle with the outer ones. Even then it would still depend on the magnitude of the shifting along the visual line how far we should approximate the situation of the centre of rotation with the situation of the point *m*.

When confining ourselves to determinations of *a*, *Mc*, and *Md* there is no reason to make these determinations for more than two values of *a*, for the values computed for the point *m* have no meaning in themselves. Indeed, KOSTER has recognized the error of this method, when he says that "it is not impossible that the eye moves either posteriorly or anteriorly, in the sense of the visual line" (p. 377). He, therefore, gives in the same table, in which he imparts the values for *ps* and *pm* (designated respectively *X* and *x*) a column with the values for the displacement *V* in the lateral position. He computes this *V* as follows ; he assumes that in the right lateral position the front point of the cornea is found, once e.g. in *x* (fig. 3). He knows this place, as he continually measures the distance *cx*. Now it is clear in connection with the foregoing, that it is just these measurings of *cx*, which after all enable us to analyse the movement to some degree (the number of observations and the accuracy are not very great). For the direction of the visual line and the situation of a fixed point of it, are known for a number of positions. The values of *cx* are not given as such, but KOSTER says he has calculated *V* as follows :

$$V = s_1x = nx - ns_1 \; ; \; nx = nc - xc = \frac{Mc}{tg\, a} - xc \text{ and } ns_1 = ps = X.$$

It might be fancied, that KOSTER made a mistake here, as it would be expected that not $ps = ns_1$, but $ps = qs_1$, considering that s_1 is to represent the anterior point of the cornea, without displacement. However from what follows, I must conclude that ns_1 was KOSTER's real intention but that, without expressing it distinctly, he relinquishes the method just described, and considers the point *m* no longer as a centre of rotation. The movement of the eye is then described as follows : (p. 382) in order to move from the central position, say, into the right lateral position, the eye first shifts over a distance *np* anteriorly, then rotates round *n*, until the visual line occupies the position ns_1 (and then $ps = ns_1$) and then again shifts in the direction of the visual line over a distance s_1n. The point *p* is then considered as a fixed point of rotation *in the orbit*, and we might conceive all eye-movements in the way just described, as consisting of two displacements combined with a rotation round the fixed point. It may be contended first of all that this view does not tally with KOSTER's own observations ; for then all visual lines must go through the centre of rotation and in KOSTER's observations this was certainly not the case. True, it might be assumed that the eye first rotates (round *p*), then shifts twice, but this does not square with the description just given. KOSTER's exposition, which is not quite clear, gives us an impression that a compromise is aimed at between his own results and those of VOLKMANN and of WOINOW. It

would follow from the experiments of these researchers which KOSTER refers to, that the visual lines intersect in one point. Such a view, which agrees with that of HERING, who assumes a centre of rotation that is supposed to be fixed in the orbit and not fixed with respect to the eye, cannot be recommended, as it is *always* possible to bring about a change of position in a plane surface by means of two shifts and one rotation, in which process it does not matter where the centre of rotation is situated.

Meanwhile there are, as I observed, sufficient data at hand to compute the position of the eye for the different angles of rotation. I proceeded as follows :

The intersection of the axes of the co-ordinates is laid in s (fig. 3) ; the one axis, which I do not call the X-axis, but the ξ-axis to prevent confusion with KOSTER's nomenclature, falls in the direction so (this direction positive), the other axis I call η-axis (positive to the right). Now let us consider the point x, of which the co-ordinates sz and zx are designated b and c in accordance with the foregoing (p. 359). We then have

$$b = sz = sn - zn = (sp - np) - zn = X - x\ tg\ \tfrac{1}{2}\alpha - (X + V)\ cos\ \alpha,$$

from which we deduce

$$b = X(1 - cos\ \alpha) - x\ tg\ \tfrac{1}{2}\alpha - V\ cos\ \alpha.$$

Furthermore $c = zx = (V + X)\ sin\ \alpha$.

So we have the general formulae :

$$b_n = X_n(1 - cos\ \alpha_n) - x_n\ tg\ \tfrac{1}{2}\alpha_n - V_n\ cos\ \alpha_n \quad . \quad . \quad . \quad . \quad (5)$$

$$c_n = (V_n + X_n)\ sin\ \alpha_n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

with the aid of which the values of b and c can be computed for every case from KOSTER's data. Now if we assume that in the displacement from every position observed, to the following, rotation takes place round one point every time, so that the expressions for b_n and c_n can be substituted in the equations (1) and (2), we find for the co-ordinates ξ and η of these centres of rotation after reduction :

$$\xi = \frac{X_{n+1}\ sin\ \tfrac{1}{2}\alpha_{n+1}\ cos\ \tfrac{1}{2}\alpha_n - X_n\ sin\ \tfrac{1}{2}\alpha_n\ cos\ \tfrac{1}{2}\alpha_{n+1}}{sin\ \tfrac{1}{2}(\alpha_{n+1} - \alpha_n)} -$$

$$- \tfrac{1}{2}(x_{n+1}\ tg\ \tfrac{1}{2}\alpha_{n+1} + x_n\ tg\ \tfrac{1}{2}\alpha_n) + \frac{(V_{n+1} - V_n)\ sin\ \tfrac{1}{2}(\alpha_{n+1} + \alpha_n)}{2\ sin\ \tfrac{1}{2}(\alpha_{n+1} - \alpha_n)}$$

$$\eta = -\frac{(X_{n+1} - X_n)\ sin\ \tfrac{1}{2}\alpha_{n+1}\ sin\ \tfrac{1}{2}\alpha_n}{sin\ \tfrac{1}{2}(\alpha_{n+1} - \alpha_n)} + \tfrac{1}{2}(x_{n+1}\ tg\ \tfrac{1}{2}\alpha_{n+1} - x_n\ tg\ \tfrac{1}{2}\alpha_n) \times$$

$$\times\ cot\ \tfrac{1}{2}(\alpha_{n+1} - \alpha_n) + \frac{(V_{n+1} - V_n)\ cos\ \tfrac{1}{2}(\alpha_{n+1} + \alpha_n)}{2\ sin\ \tfrac{1}{2}(\alpha_{n+1} - \alpha_n)}.$$

In this way the data of the following table have been obtained (to the right the angles are positive, to the left negative).

Left eye: horizontal movement; opened normally; lengths in mm.

α	$\delta_{(\alpha_{n+1}-\alpha_n)}$	X	x	V	b	c	ξ	η
— 45°		13.2	1.5	0.1	4.4	— 9.4		
	15°						13.5	— 0.4
— 30°		13.8	1.1	— 0.3	2.4	— 6.8		
	10°						15.4	— 3.2
— 20°		14.6	— 2.3	— 1.6	2.0	— 4.4		
	10°						16.7	— 5.4
— 10°		14.4	— 13.7	— 3.4	2.4	— 1.9		
	10°						12.1	12.6
0°				0	0	0		
	10°						16.5	10.9
10°		14.4	— 13.7	3.1	— 1.6	3.0		
	10°						14.4	0.1
20°		14.6	— 2.3	2.3	— 0.9	5.8		
	10°						8.7	— 3.4
30°		13.8	1.1	0.8	0.9	7.3		
	15°						10.9	0.8
45°		13.2	1.5	0.5	2.9	9.7		

The difference between the points now found as centres of rotation and the points m is distinct. It is striking that, whereas the centres of rotation in a rotation from the zero-position to $+10°$ and $-10°$, are both situated more than 10 mm to the right, the corresponding point m lies nearly 14 mm to the left. It would be wrong to ascribe these differences to the inaccuracy of the observations: the fact that the points have another meaning is responsible for them.

KOSTER's results for the wide-opened eye (in which, as pointed out already by J. J. MÜLLER, the bulbus protrudes) have been obtained in the same way. The results are corresponding; they have been tabulated in the

second table. According to KOSTER the point s now lies 0.8 mm to the front; I have assumed that the data of his table relate to this new point s, so that my results are also numerical values concerning the new central-position of the eye.

Finally I have made a graph of most data of the first table. (The line $y_4\ y_3 \ldots . x_3\ x_4$ illustrates the path of the cornea, for the positions from —45° to +45°; Q_4, $Q_3 \ldots . P_3$, P_4 represent the corresponding centres of rotation, while m_1, m_2, m_3 and m_4 are the centres of KOSTER for rotations over angles of respectively $\mp 10°$, $\mp 20°$, $\mp 30°$, en $\mp 45°$.)

Whether the flexion in the curve at y_1 points to a discontinuity in the movement, or whether it is caused by the inaccuracy or the small number of observations, I will not venture to decide. It is obvious what difficulties one would meet, when working out these data in the equations (3) and (4); I, therefore, abstained from that operation.

Left eye: horizontal movement; wide-opened; lengths in mm.

α	$\delta_{(\alpha_{n+1}-\alpha_n)}$	X	x	V	b	c	ξ	η
— 45°		13.9	2	0.3	4.7	—10.0		
	15°						15.8	0.4
— 30°		13.7	1.5	— 0.1	2.3	— 6.8		
	10°						15.5	— 2.5
— 20°		14.3	— 1.7	— 1.3	1.8	— 4.4		
	10°						12.6	11.7
— 10°		13.8	— 16.6	— 0.4	— 0.8	— 2.3		
	10°						12.9	— 6.0
0°				0	0	0		
	10°						16.3	12.3
10°		13.8	— 16.6	3.6	— 1.9	3.0		
	10°						12.3	— 6.2
20°		14.3	— 1.7	1.3	— 0.1	5.3		
	10°						10.3	— 0.4
30°		13.7	1.5	0.4	1.1	7.1		
	15°						11.8	— 1
45°		13.9	2	— 0.4	3.5	9.5		

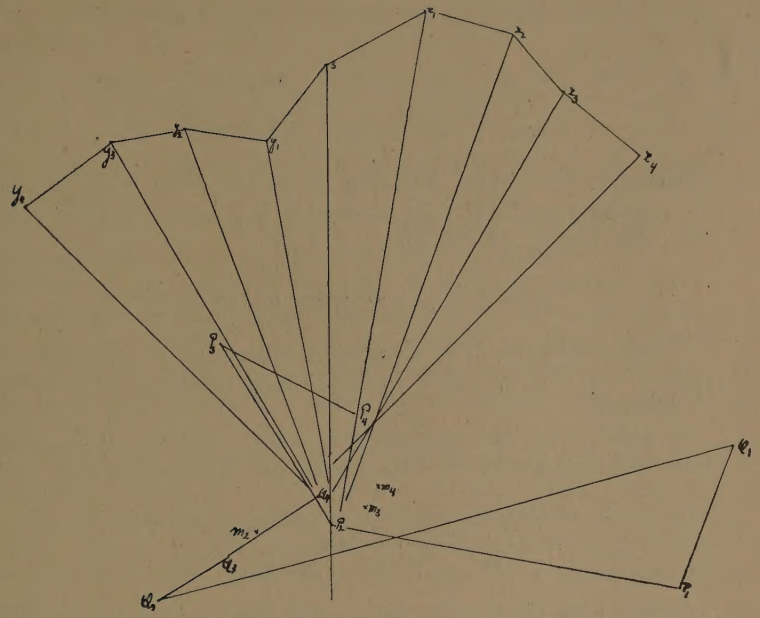


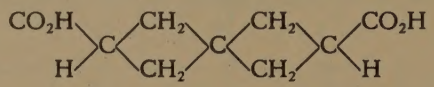
Fig. 4. (Scale about 5/1).

Chemistry. — *Optical resolution of a spirocyclic compound of the allene type.* By H. J. BACKER and H. B. J. SCHURINK. (Communicated by Prof. F. M. JAEGER.)

(Communicated at the meeting of April 28, 1928).

Recent publications [1]) on spirocyclic compounds of the asymmetric allene type xyC : C : Cxy induce us to publish some results of our investigations in this field.

In order to test the stereochemical theory, that such spiranes should exist in enantiomorphs, we have studied *spiroheptanedicarboxylic acid :*



FECHT [2]) has obtained this acid in small yield, starting from tetrabromopentaerythritol and malonic ester.

After having improved the preparation until the yield was about 80 %, we attempted the resolution by means of the dibrucine salt, which crystallises with six molecules of water in short prisms and decomposes at about 135°.

This brucine salt, recrystallised carefully from water and decomposed by ammonia, gives an optically active ammonium salt. The rotation is feeble and reaches its maximum value after about 5 crystallisations.

The rotatory power is given in the following table for different wave lengths.

Rotatory dispersion of ammonium spiroheptanedicarboxylate.

$\lambda(\mu\mu)$	656.3 (C)	589.5 (D)	546.3 (Hg)	486.1 (F)
$[\alpha]$	+ 0°.11	0°.13	0°.15	0°.19
[M]	+ 1°.9	2°.3	2°.6	3°.4

The free acid being only little soluble in water cannot be studied accurately in aqueous solution. Its ethereal solution shows a positive rotation : $[M]_D = +1°.9$.

We believe that this is the first observation of an optically active spirane of this type, whose resolution is completely reproducible.

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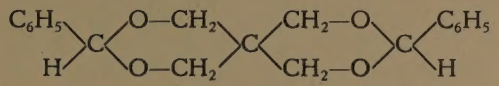
[1]) PFEIFFER and BACKES, Ber. **61**, 434 (1928); BÖESEKEN and FELIX, Ber. **61**, 787 (1928).

[2]) Ber. **40**, 3888 (1907).

Further we studied a compound of analogous structure, the *dibenzalpentaerythritol :*



READ [1]) has sought already in 1912 for optical activity in the case of this and related compounds.

BÖESEKEN and FELIX [2]) publish to have observed for this compound on one occasion an optical activity, which could not be reproduced.

Our results, already obtained in 1927, may be shortly summarized.

The dibenzal compound, prepared under different conditions, had always the same melting point (162°) and the same chemical properties. Thus the occurrence of a cis-trans isomerism, which might be possible in the case of a pyramidal distribution of the valences of the central carbon atom, is improbable.

The substance was obtained in large hexagonal crystals with edges up to half an inch in length.

The Röntgen analysis, made in the inorganic chemical laboratory by Mr. VAN MELLE, showed that the crystal has a three-fold screw axis, perpendicular to the basal plane. Thus dextro- and laevorotatory crystals should be possible.

Indeed both kinds of crystals were obtained. The rotation could not be measured accurately with our polarisation apparatus ; the value is about $a_D = \pm 2°/mm$.

The Röntgen analysis of dibenzalpentaerythritol, the results of which will be published elsewhere, has proved, that the molecules in the crystal possess three mutually perpendicular two-fold axes, according to the symmetry of an orthorhombic bisphenoïde. Thus the central carbon atom has a tetrahedral (not a pyramidal) distribution of its valences and the phenyl groups must be placed in the long axis of the molecule, for instance :

The optical activity of the crystals is caused by the asymmetric distribution of the molecules.

None of the crystals, when dissolved in alcohol, ethyl acetate or chloroform, has shown a trace of optical activity.

Groningen, April 1928.

Organic chemical laboratory of the State University.

[1]) J. chem. Soc. **101**, 2090 (1912).

[2]) Loc. cit.